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AERODYNAMIC HEATING AND STRUCTURAL
TEMPERATURE RESPONSE,

July, 1963

by Stanley C. Browning ,

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SYMBOLS AND ABBREVIATIONS

A   Area, sq. ft.

a   Velocity of sound, ft/sec.

C   Heat capacity, Btu/deg F

C_f Local friction factor

C_p Specific heat, Btu/lb-deg F

d   Depth of element (generally taken as unity), ft.

b   Conductance across an interface, Btu/sec-deg F

F_a Configuration or angle factor for radiation

F_e Emissivity factor for radiation

g   Acceleration due to gravity, ft/sec^2

H   Altitude, ft.

h   Local heat-transfer coefficient, Btu/sec-sq ft-deg F

J   Mechanical equivalent of heat, ft/lbs/Btu

K   Conductance, Btu/sec-deg F

k   Thermal conductivity, Btu/sec-sq.ft. (deg F/ft)

K.E.   Kinetic energy, ft-lbs.

m   Mass, $lb-sec^2/ft.$

Ma   Mach Number

Nu   Nusselt Number

Pr   Prandtl Number

q   Heat flow per unit time and area Btu/sec-sq.ft.

R   Gas constant

r   Temperature recovery factor

Re   Reynolds Number

S   Geometric factor

St   Stanton Number

T	Absolute temperature, deg R
t	Temperature, deg F
T_{bl}	Boundary layer (or recovery) temperature, deg R
T_o	Stagnation temperature, deg R
T_∞	Ambient or free stream temperature, deg R
u	Velocity component parallel to wall, ft/sec.
U	Local velocity at any point (x-direction), ft/sec.
V	Volume, cu.ft.
v	Velocity component normal to wall, ft/sec.
x	Distance from leading edge, ft.
y	Distance normal to wall, ft.
γ	Ratio of specific heats
δ	Boundary layer thickness, ft.
θ	Time, sec.
$\Delta\theta$	Finite increment of time, sec.
μ	Viscosity, lb/ft-sec.
ρ	Density, lb/cu.ft.
Σ	Summation
σ	Stefan-Boltzman constant, Btu/hr-sq.ft-R^4
τ	Shearing stress, lb/sq.in.

Subscripts and Superscripts

c	Convection
k	Conduction
o	Stagnation point or a nodal designation
r	Recovery or radiation
s	In the stream just outside the boundary layer

w   At wall surface

1,2,3----i Nodal designation

'   End of a time interval

*   Reference condition

ABSTRACT

A simplified method for determining the aerodynamic heating that occurs on vehicles traveling at high speeds is presented. Relations are submitted which permit the calculation of the temperature of the boundary layer and the heat transfer coefficient as a function of Mach number and altitude. Both laminar and turbulent flow within the boundary layer are considered and the method has been programmed for use with ABL automatic digital computation equipment. Equations for predicting structural temperatures resulting from aerodynamic heating are also presented and are incorporated into the over-all program in order that the effects of surface temperature on the aerodynamic heat input can be taken into account.

The program can also be applied to problems other than those specifically having to do with aerodynamic heating. Heat flow in two directions accounting for all modes of heat transfer and the variation of physical properties with temperature may be considered. Also, various materials in series along with resistances at their interfaces may be studied concurrently.

The major limitations and range of applicability are explained, and typical examples are presented to clarify the procedures.

1

INTRODUCTION

A vehicle traveling at supersonic speeds within the atmosphere generates severe thermal environments by its high-speed motion. The boundary layer concept proposed by Prandtl has proved to be a very valuable tool in studying the heating effects due to flight under these conditions. The particles of fluid in a thin layer near the body are acted upon by shearing, or viscous, frictional forces and, within this layer, the relative velocity between the vehicle and ambient fluid is slowed down to zero at the surface. The kinetic energy of the air particles then appears as heat energy which causes the boundary layer temperature to rise. Some of the heat from the boundary layer flows into the surface of the vehicle, the amount increasing rapidly with increasing flight speed. This convective heat-transfer process is commonly referred to as aerodynamic heating.

The obvious result of aerodynamic heating is, among other things, a weakening of the structure if it is not adequately protected by insulation or some means of cooling. Aerodynamic heating is of particular interest to the solid-propellant rocket designer because prolonged exposure to such heating is likely to cause a failure at the propellant-to-chamber bond surface in addition to an over-all weakening of the basic load carrying structure. It is, therefore, necessary to establish a means of evaluation of aerodynamic heating so that proper consideration can be given to the structural design and heat protection system requirements for rocket motors.

The purpose of this report is to describe a method by which heat transfer coefficients and boundary layer temperatures can be calculated and in turn used for determining structural response to aerodynamic heating. Both laminar and turbulent flow within the boundary layer have been considered and the method has been programmed for use with ABL automatic digital computation equipment.

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The procedure is an intentionally simplified one designed to provide answers to our current problems and should not be expected to yield satisfactory results for all conditions of high-speed flight. Major limitations and the range of applicability are explained. Also, typical examples are presented to clarify the procedures involved in solving several types of aerodynamic heating and heat transfer problems.

In general, the method can be used for flight speeds of less than Mach 10 and altitudes below 250,000 feet. Very high boundary layer temperatures result at high Mach numbers; consequently the oxygen molecules in the air begin to dissociate. At extremely high temperatures ($\geq 10,000°R$) ionization becomes significant. Although methods are available for treatment of these phenomena, they have not been included in this presentation.

Interaction of the boundary layer and shock wave near the leading edge of a hypersonic vehicle has an effect on the aerodynamic heating. Consideration of this effect was not considered necessary for our purposes. Also, radiation to or from the boundary layer has been neglected. It should be pointed out that most of our (ABL) studies are concerned with relatively low, compared to atmospheric re-entry for example, velocity flight at low altitudes and the limitations of the present program need not concern us greatly. For example, in the POLARIS A3 trajectory the results may appear questionable in the final seconds of operation, due to the limitations previously cited; however, the peak aerodynamic heating and maximum skin temperatures, which are usually the items of major concern, are predicted earlier in the trajectory. In other words, by the time such effects as dissociation and ionization must be taken into account, the structure is losing energy at a rate greater than it is gaining it. Furthermore, in these late stages of heating when the coefficient is small, structural temperatures are essentially unaffected by its changing.

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The surface temperature and film coefficient of heat transfer between the boundary layer and an aerodynamically heated structure are very closely interrelated; therefore, it was necessary to include a procedure for determining the temperature distribution within the structure. Owing to its inherent simplicity and speed of calculation, the two-dimensional, finite-difference technique presented by Dusinberre[1] was chosen for this purpose. Both the heat-transfer and aerodynamic-heating phases of the resulting program can be used independently; however, only an approximate value of the film coefficient can be obtained when the surface temperature is not computed at the same time.

AERODYNAMIC HEAT INPUT

Equations representing the flow of a compressible, heat conducting and viscous fluid (such as air) can be derived from the principles of conservation of energy, momentum and mass, together with an equation of state. These equations can be simplified by introducing the boundary layer concept, in which changes along the length of the layer are considered small compared to the changes across it. For this simple case the equations, based on energy momentum and mass balances, are:

$$\rho \, C_p \left(u \, \frac{aT}{ax} + v \, \frac{aT}{ay} \right) = \frac{a}{ay} \left(k \, \frac{aT}{ay} \right) + \mu \left(\frac{au}{ay} \right)^2 \tag{1}$$

$$\rho \left(u \, \frac{au}{ax} + v \, \frac{au}{ay} \right) = \frac{a}{ay} \left(\mu \, \frac{au}{ay} \right) \tag{2}$$

$$\frac{a \, \rho u}{ax} + \frac{a\rho \, v}{ay} = 0 \tag{3}$$

The equation of state is:

$$\rho \, RT = \text{Constant} \tag{3a}$$

Consider Equation (1) which describes the conservation of energy. On the left hand side, the terms in parentheses represent convective heat transfer; on the right hand side, the first term denotes conductive heat transfer and the second term is considered a "forcing function" which is proportional to $\frac{au}{ay}$. For a small boundary layer thickness, δ, and large velocities, u, the velocity gradient within the layer, $\frac{au}{ay}$, can be considered approximately equal to $\frac{u}{\delta}$. Therefore,

$$\left(\frac{au}{ay} \right)^2 \approx \left(\frac{u}{\delta} \right)^2$$

As the velocity u increases $\left(\frac{u}{\delta} \right)^2$ assumes an increasingly important role. This forcing function represents the rate of dissipation of mechanical energy into heat and is, therefore, the source of aerodynamic heating.

The above equations can be put into dimensionless form by grouping the variables into parameters involving two or more variables. The advantage of this procedure is that for geometrically similar cases the equations have the same values for the parameters, and the solutions, in dimensionless form, are equal.

Equations (1), (2), and (3) are not in useful form since many of the terms are difficult to evaluate. A major effort in gas dynamics investigations in recent years has been the evaluation of derived parameters, such as the familiar convective film coefficient, for suitably representing the actual process.

Although there appears to be almost as many aerodynamic heating analysis methods as there are practicing aerodynamicists, the following methods have been selected and developed for purposes of simplicity and engineering application. These procedures, and, consequently, the associated computer program can be modified as the need for more general results arises. In general, results obtained by the methods reported herein will be valid to speeds of Mach 10 and altitudes of less than 250,000 feet. This range may be extended by incorporating relationships to account for dissociation and ionization.

The methods and equations given below are based upon the "Reference Temperature Method" of Eckert[2]. The procedures are based on flow past a flat plate at zero angle of attack; however, the same relations can also be used to approximate conditions on slender bodies in supersonic flow when the region near the leading edge is excluded. It is assumed that both the heat transfer and skin friction can be calculated from incompressible flow relations provided all temperature dependent air properties are evaluated at an appropriate reference temperature which lies somewhere between the wall and the boundary layer or recovery temperature.

Procedures for computing the boundary layer or recovery temperature, the reference temperature and the film coefficients corresponding to both laminar and turbulent flow are presented in the following subsections.

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Dimensionless Flow Parameters

The following dimensionless parameters will be used to simplify the analysis:

$$Re = \frac{\rho\, U_s x}{\mu} \tag{4}$$

$$Ma = \frac{U_s}{a_s} \tag{5}$$

The property values μ and ρ will be introduced as occurring at different locations (wall, stream, etc.) whereas the Mach number Ma is always based on the sound velocity a_s at the stream temperature.

The local shearing stress $\mathcal{T}_w'$ which the flow exerts on the wall surface is expressed by a dimensionless friction factor C_f defined by the following equation:

$$\mathcal{T}_w' = C_f\, \rho\, \frac{U_s^2}{2} \tag{6}$$

The temperature recovery factor is defined as

$$r = \frac{T_{b1} - T_{\infty}}{T_o - T_{\infty}} = \frac{T_{b1} - T_{\infty}}{U_s^2/2\, C_p} \tag{7}$$

and is a function of the Prandtl number

$$P_r = \frac{C_p \mu}{k} \tag{8}$$

The heat transfer coefficient is ordinarily expressed in terms of the Nusselt number

$$Nu = \frac{hx}{k} \tag{9}$$

or the Stanton number

$$St = \frac{h}{\rho\, C_p V_s} \tag{10}$$

It can be shown that certain of the above dimensionless parameters are related as follows:

$$St = \frac{Nu}{Re\, Pr} \tag{11}$$

Identification of each of the individual symbols is given in the list of Symbols and Abbreviations.

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Boundary Layer Temperature

For one pound of air slowed down from a given U to a zero velocity, the kinetic energy given up by the fluid is

$$K.E. = \frac{mU^2}{2} = \frac{U^2}{2g} = \frac{U^2}{2gJ} \tag{12}$$

where J is Joules constant representing the mechanical equivalent of heat and is equal to 778 ft-lbs/Btu. For complete recovery, the temperature rise of the air from this kinetic energy is

$$T_0 - T_\infty = \frac{U^2}{2gJC_p} \tag{13}$$

Therefore,

$$T_0 = T_\infty + \frac{V^2}{2gJC_p} \tag{14}$$

which can be shown to equal[3]

$$T_0 = T_\infty \left[1 + \frac{\gamma-1}{2} M_a^2 \right] \tag{15}$$

All of the energy is not recovered, however, and Equation (15) must be modified through the use of the recovery factor previously defined.

$$r = \frac{T_{b1} - T_\infty}{T_0 - T_\infty} \quad \frac{T_{b1} - T_\infty}{\frac{U^2}{2C_p}} \tag{7}$$

Repeated

The recovery factor can be approximated by

$$r = \sqrt[3]{Pr} \tag{16}$$

Substituting the recovery factor into Equation (15)

$$T_{b1} = T_\infty \left[\frac{\gamma-1}{2} M_a^2 \gamma \right] \tag{17}$$

Based on experimental data

$$r = 0.85 \quad \text{for laminar flow}$$

$$r = 0.9 \quad \text{for turbulent flow}$$

For air, $\gamma = 1.4$, and Equation (17) reduces to

$$T_{b1} = T_\infty (1 + 0.17 M_a^2) \tag{18}$$

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for laminar flow, and

$$T_{b1} = T_\infty \ (1 + 0.18 \ M_a^2) \tag{19}$$

for turbulent flow.

Heat Transfer Coefficients

The mechanism of heat transfer from the boundary layer to a surface can be written:

$$q = h \ A \ (T_w - T_{b1}) \tag{20}$$

Thus, in estimating the heat transfer rate to a structure, it is necessary to know the heat transfer coefficient beforehand.

Laminar Flow: Assuming constant property values for the fluid, it has been determined[4] that for laminar flow across a flat plate with zero pressure gradient, the local skin friction factor is

$$C_f = 0.664 \ Re^{-\frac{1}{2}} \tag{21}$$

and the local Nusselt's number is

$$Nu = 0.332 \ (Re)^{\frac{1}{2}} \ (Pr)^{1/3} \tag{22}$$

By substituting Equations (21) and (22) into Equation (11), the Stanton number for laminar flow is given by

$$St = \frac{C_f}{2} \ (Pr)^{-2/3} \tag{22a}$$

The Reynolds analogy[4] between skin friction and heat transfer is expressed as

$$St = \frac{C_f}{2} \tag{23}$$

This relationship for laminar flow is a reasonable first approximation for subsonic flow since the Prandtl number is approximately 1 and it doesn't make a great deal of difference where the fluid properties are evaluated. At higher velocities, however, with increasing boundary layer temperatures, the Reynolds analogy will be in error by an amount approximately equal to $(Pr)^{-2/3}$ and the fluid property values become increasingly important.

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For laminar flow, the heat-transfer coefficient, h, can now be written as

$$h = 0.332 \frac{k}{x} (Re)^{\frac{1}{2}} (Pr)^{1/3} \tag{24}$$

or

$$h = 0.332 \, kx^{-1/2} \left(\frac{Re}{x} \right)^{\frac{1}{2}} (Pr)^{1/3} \tag{25}$$

For supersonic flight, we cannot assume constant fluid properties; therefore in evaluating the parameters entering into the solution of Equation (25), the variable fluid properties must be taken at some specific temperature. If the temperature chosen is the boundary layer or recovery temperature, we find that the friction factor decreases with increasing Mach number and increasing ratio of T_w/T_∞. On the other hand, if the temperature chosen is the wall temperature, the opposite change in skin friction factor is noted. From this it might logically be inferred that some intermediate temperature can be found at which the variation of friction factor with Mach number and T_w/T_∞ vanishes. It is on this reasoning that the Reference Temperature Method is based.

Rubesion and Johnson[5] have shown that the reference temperature can be accurately expressed by the following equation:

$$T^* = T_\infty \left[1 + 0.032 \, M_a^2 + 0.58 \left(\frac{T_w}{T_\infty} - 1 \right) \right] \tag{26}$$

This expression for reference temperature can be rewritten as

$$T^* = T_\infty \, (0.42 + 0.032 \, M_a^2) + 0.58 \, T_w \tag{27}$$

The last factor in Equation (27) contains the term T_w which is what we ultimately want to determine. However, the wall temperature T_w must be found from a heat transfer analysis of the structure. The logical way to approach this problem, and the one used in this instance, is to divide a given missile trajectory into increments of time and to calculate heat transfer and surface temperature at the end of a time increment based on a value of T_w, and thus h, evaluated at a surface temperature at the beginning of the time increment. The newly computed value of T_w is then used to determine a new reference temperature and film coefficient to be used over the next increment of time.

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For speeds below M5, the difference between boundary layer temperature and wall temperature is not too great, so with small error we can substitute Equation (18) into Equation (27).

$$T^* = T_\infty \ (0.42 + 0.032 \ M_a^2) + 0.58 \ \left[T_\infty \ (1 + 0.17 \ M_a^2) \right]$$

and

$$T^* = T_\infty \ (1 + 0.131 \ M_a^2) \tag{28}$$

Thus, in certain situations we are able to determine T^*, with a reasonable degree of accuracy, independent of wall temperature. Equation (28) has been programmed for use in calculating the film coefficients for instances when the problem is such that the wall temperature cannot be determined; i.e., when the structural physical properties have not been set, such as in preliminary phases of rocket design. When possible, the more desirable method of determining the reference temperature, Equation (27), is used. The viscosity, density and conductivity of the fluid are found at this temperature and substituted into Equation (25) for computing the heat transfer coefficient for laminar flow.

Turbulent Flow: For laminar flow the calculation of friction and heat transfer by theoretical means is usually recognized as quite reliable. Such is not the case for turbulent flow, however. At best, turbulent flow theories are semi-empirical and have limited ranges of validity. Exact solutions of the boundary layer equations, based on the present day understanding of the problem, are virtually impossible. This necessitates heavy reliance on experimental measurements to obtain friction and heat transfer data. The various theories which have been published deviate considerably from each other, especially at higher Mach numbers. The following procedure agrees with experimental results within the investigated range.

For a flat plate at zero angle of attack and zero pressure gradient, the following formula gives good agreement with experimental data up to $Re = 10^7$.[6]

$$\frac{C_f}{2} = 0.0296 \ Re^{-0.2} \tag{29}$$

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Substituting Equation (29) into Equation (22a) for the Stanton number

$$St = \frac{Nu}{Re\ Pr} = 0.0296\ (Re)^{-0.2}\ (Pr)^{-2/3} \tag{30}$$

Therefore,

$$Nu = \frac{hx}{k} = 0.0296\ (Re)^{0.8}\ (Pr)^{1/3} \tag{31}$$

Solving for the film coefficient

$$h = 0.0296\ \frac{k}{x^{0.2}}\ \left(\frac{Re}{x}\right)^{0.8}\ (Pr)^{1/3} \tag{32}$$

As in the case of laminar flow, the fluid properties must be evaluated at a reference temperature. The appropriate reference temperature is derived in a manner analogous to that for laminar flow. Substituting Equation (19) into Equation (27),

$$T^* = T_\infty\ (0.42 + 0.032\ M_a^2) + 0.58 \left[T_\infty\ (1 + 0.18\ M_a^2)\right] \tag{33}$$

which simplifies to

$$T^* = T_\infty\ (1 + 0.137\ M_a^2) \tag{34}$$

Equations (25) and (32) indicate the form of solution for the heat-transfer coefficient. As noted earlier, the solutions are intended to be applicable to a flat plate only. For laminar flow past a cone, the heat transfer computed from Equations (25) and (20) should be multiplied by $\sqrt{3}$. In the case of turbulent flow past a cone, multiply Re by 2 and proceed as for a flat plate.

The work involved in picking property values and evaluating the quantities involved in the solution of the equations for the film coefficient can become most laborious. This makes hand computations over a trajectory lasting for any appreciable time highly impractical. However, the problem can be handled very quickly by the use of a high-speed digital computer. All the above equations that are necessary for the solution, along with data and necessary equations for property evaluation, have been programmed on the IBM 7074 computer now in use at ABL. Details pertaining to air property evaluation and computer programming are discussed in later sections of this report.

Approximate Method for Estimating Heat Transfer Coefficients

Reference (7) contains a compilation of working curves based, with some modification, upon the reference temperature method of Eckert.[2] This method can be useful in making computations by hand if an accurate estimate of the wall temperature can be made. As was noted earlier, it is possible in the low-speed regime to approximate the wall temperature with the boundary layer temperature.

Another very simple method was learned by the writer while with the Martin Company and is included for very rough estimations only. An equation giving the heat transfer index has been prepared in nomograph form for the convenience of the designer and has been included as Figure 1. The following procedures are provided to explain the application of the nomograph:

1. Plot altitude and Mach number as a function of time for the trajectory under consideration.

2. Using Standard Atmosphere Charts, plot ambient temperature as a function of time.

3. Calculate reference temperature from Equation (28) and plot as a function of time.

4. For a particular time, compute the ratio of reference temperature to ambient temperature. With this ratio plus the Mach number and altitude at that time, enter Figure 1 to determine $hx^{0.2}$ where x is the distance from the leading edge or nose. Divide the index by $x^{0.2}$ and plot on a curve giving h as a function of time. The h values can then be used to make a crude estimate of heat transfer to the structure.

Atmospheric Properties

Since all aerodynamic heating calculations are dependent upon atmospheric temperature, Figure 2 was included to show the variation of ambient temperature with altitude assuming either the NACA standard atmosphere[8] or the ANA standard
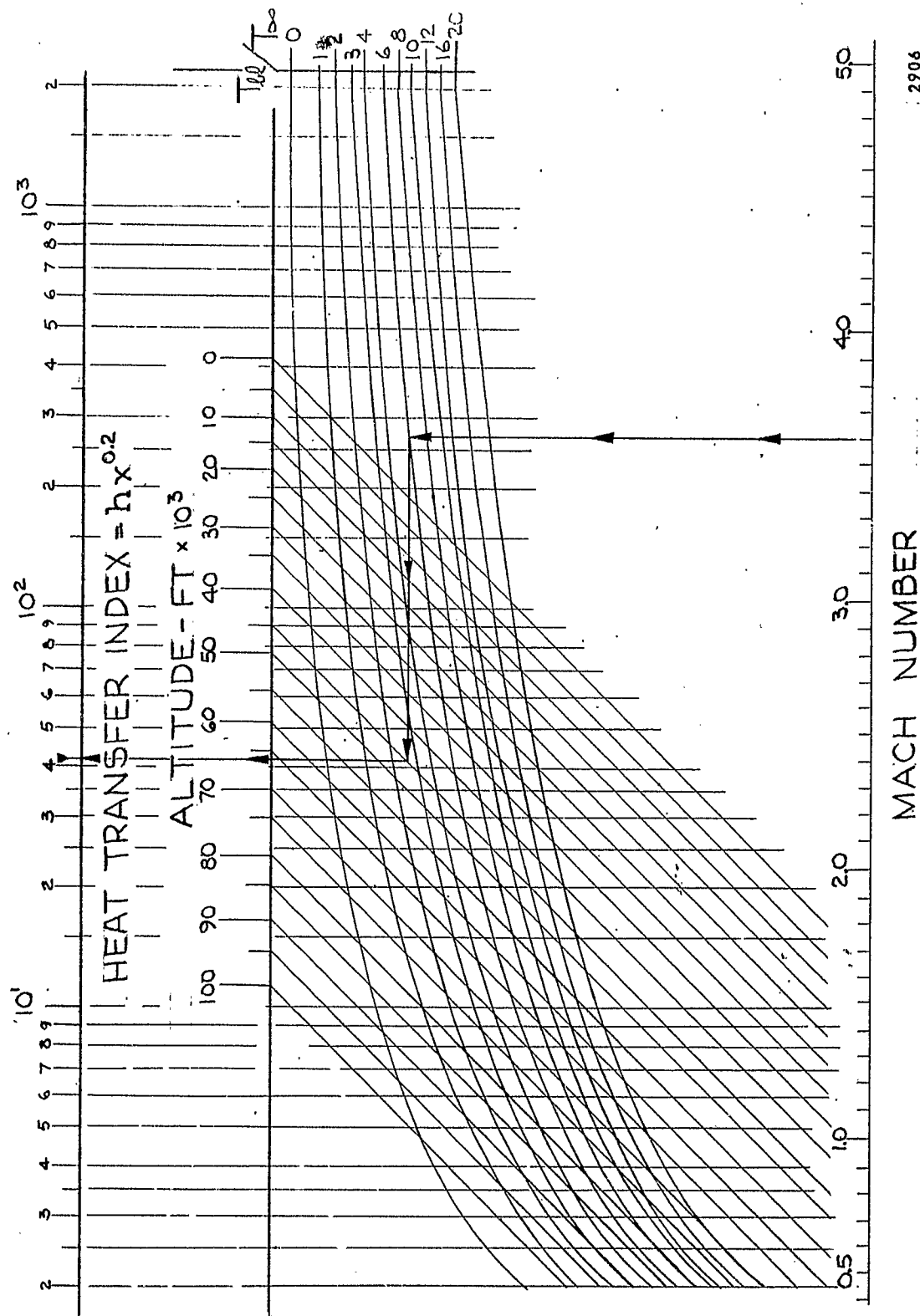
FIGURE 1

Nomogram for Approximate Calculation of Aerodynamic Heat Transfer Coefficient

MACH NUMBER

HEAT TRANSFER INDEX = $hx^{0.2}$

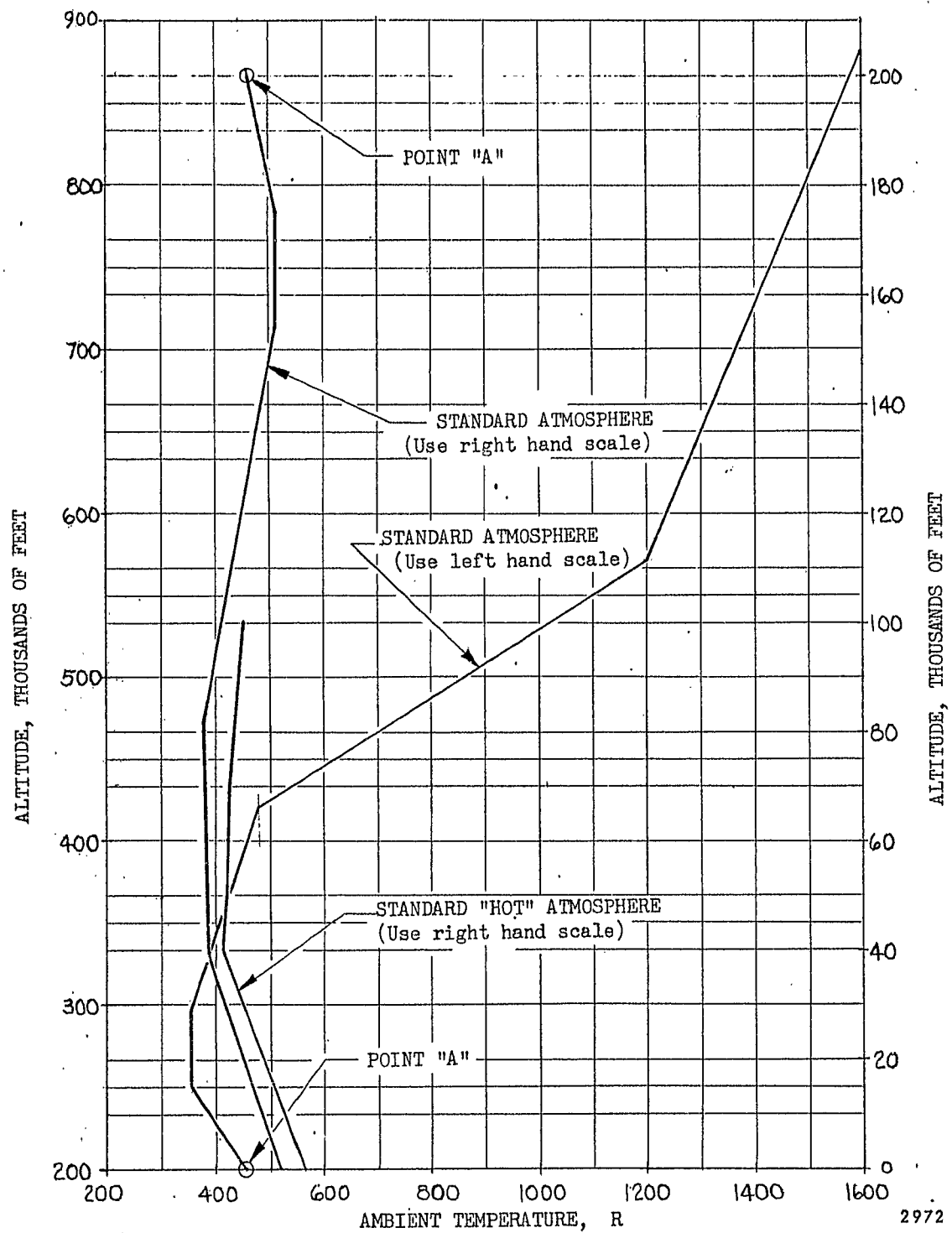ALTITUDE - FT $\times 10^3$

2906

13A

FIGURE 2
Variation of Ambient Temperature with Altitude

13B

hot atmosphere. To simplify the computer program, the temperature-altitude curves were conveniently represented by algebraic equations.

The variation of air density with altitude was predicted from equations which had been in use previously in related studies[9] at ABL.

Other air properties necessary for calculating the aerodynamic heat transfer coefficient are thermal conductivity, viscosity and specific heat. Also, since the quantitative values for each of these properties changes with changing temperature, and, since they must be determined at a variable reference temperature, each must be expressed as a function of temperature. This also holds for the air density; consequently the density of air at a given altitude and corresponding ambient temperature must be modified when solving for the density at the reference temperature. All of the equations relating properties and temperature which were utilized in the program, along with the reference from which they were taken, are given in the Equation Summary and will not be repeated here.

The velocity input to the aerodynamic heating equations must be in terms of Mach number. Since the velocity in feet per second must be divided by the speed of sound in determining Mach number, the sonic velocity must be known for the altitude in question. The atmospheric temperature is available from Figure 2 and the sonic velocity can be calculated from the well known expression

$$a = 49.02 \ (T_{\infty})^{\frac{1}{2}} \tag{35}$$

The sonic velocity up to an altitude of 100,000 feet can be read from Figure 3.
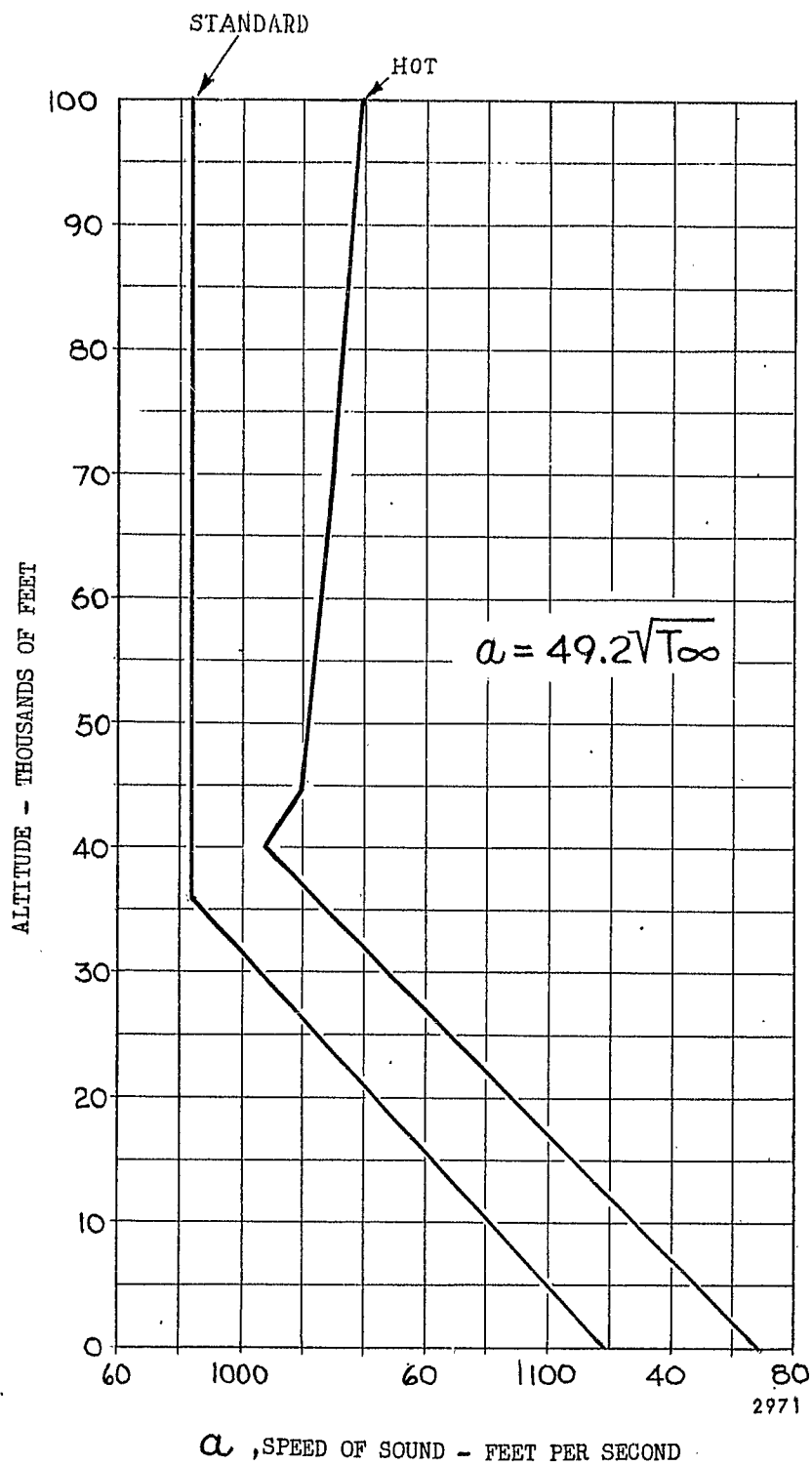
FIGURE 3
Velocity of Sound to an Altitude of 100,000 Feet

NUMERICAL METHOD FOR TRANSIENT HEAT-FLOW ANALYSIS

The general numerical method of Dusinberre was selected as the most simple and convenient procedure for determining structural response to aerodynamic heating. The development of the fundamental equations is briefly described below. Only the detail considered necessary for a general understanding of the problem has been included in this report; those wishing a more thorough treatment of the subject are referred to Reference (1).
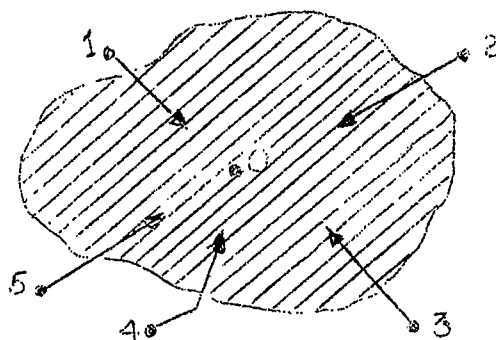
Figure 4

Consider the point 0 as shown above. The temperature at that point is taken as representative of a certain region which includes the point, and the heat flow at the boundary is taken as the summation of flows from surrounding regions. The transfer can take place by any of the three modes of heat transfer; that is, convection, conduction or radiation. In the general case, heat can also be generated within the region itself; however, for this study it is not considered necessary to take this effect into account. The net heat flow into the region must be stored therein, thus causing the temperature of node 0 to rise.

Conduction

For a conductive interchange from some node i, representative of a region adjacent to the region containing 0, the rate of heat transfer q_k to node 0 is given by

$$q_k = K_k (t_0 - t_i) \tag{36}$$

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where K_k is the conductance along the path i to 0 and t_0 and t_i are the temperatures at nodes 0 and i, respectively.

In general, for a single material

$$K_k = kS \tag{37}$$

where S is a geometric factor depending on the particular subdivision of the system. In this report only squares, rectangles, and annular networks are considered; the S values corresponding to each of these subdivisions are given in Table I. The program is not limited to a subdivision consisting of the above geometric figures, however. For situations in which a triangular network appears necessary, the geometric factors can be calculated from expressions given in Reference (1). The procedure may also be obtained from the author upon request.

For a single material the conductance K_k is obtained by multiplying the geometric factor by the material thermal conductivity; however, for materials in series, air gaps and contact resistances the rules for addition of conductances must be applied. These procedures are explained in almost any good text on heat transfer.

Convection

For the convective mode of heat transfer from i to 0, the flow rate, q_c is

$$q_c = K_c (t_0 - t_i) \tag{38}$$

In this case h is the convective film coefficient and A is the cross-sectional area normal to the flow path. All other terms are as given for conduction.

Unlike the thermal conductivity k, it will be necessary to have h vary with time and surface temperature. Furthermore, it will often be necessary to calculate h for each time increment as the wall surface temperature changes. Procedures for calculating h in a special application are given under the section of this report entitled "Aerodynamic Heat Input."
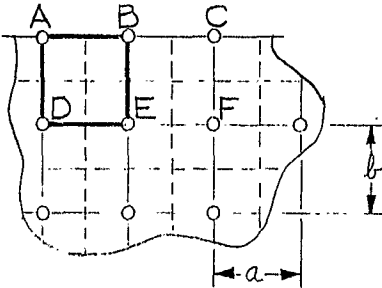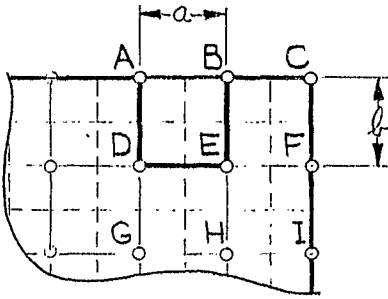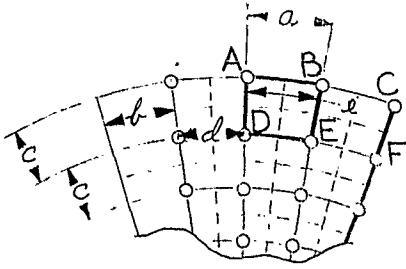
Radiation

For the case of radiation between bodies

$$q_r = AF_a F_e \sigma (T_0^4 - T_i^4) \tag{39}$$

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TABLE I
Calculation of Geometric Coefficient S

Note: The area A, normal to the heat flow path, is bounded by the perpendicular
bisectors of adjacent heat flow paths. The geometric coefficient S is
obtained by dividing A by the length of the corresponding path.

NETWORK	DATA	S
SQUARE (a = b)	A B D E	$S_{AB} = \dfrac{b}{2a} = \dfrac{1}{2}$ $S_{DE} = \dfrac{b}{a} = 1$
RECTANGLE (a ≠ b)	A B B E D E C F	$S_{AB} = \dfrac{b/2}{a}$ $S_{BE} = \dfrac{a}{b}$ $S_{DE} = \dfrac{b}{a}$ $S_{CF} = \dfrac{a/2}{b}$
ANNULAR SEGMENTS	A B B E D E C F	$S_{AB} = \dfrac{c/2}{\ell}$ $S_{BE} = \dfrac{\ell}{c}$ $S_{DE} = \dfrac{c}{d}$ $S_{CF} = \dfrac{b/2}{c}$

F_a is usually called a configuration factor and is a function of angles of radiation, area and distances between radiating surfaces. F_e is an emissivity factor to account for the departure of the two surfaces from complete blackness and is a function of the emissivities as well as the configuration of the surfaces. σ is the Stefan-Boltzmann constant.

$$\sigma = 1730 \times 10^{-12} \; Btu/(hr.)(sq.ft.)(°R)^4$$

The temperatures, represented by T_o and T_i must be expressed on the absolute scale.

It is convenient to represent radiation heat transfer in a manner similar to convection and conduction in which case

$$q_r = K_r \; (T_o - T_i) \tag{40}$$

hence,

$$K_r = h_r A$$

with

$$h_r = F_a F_e \; \sigma \; (T_o^3 + T_o^2 T_i + T_o T_i^2 + T_i^3) \tag{41}$$

The configuration factor and emissivity factor in the above equations were combined into a single term for computer programming.

Since this study is primarily oriented toward aerodynamic heating analysis, it will usually be assumed that radiation is from the aerodynamically heated surface to a surface at 0° temperature. This is the usual assumption for radiation from a body traveling through space. In this case

$$q_r = \sigma \; A T_o^4 \tag{42}$$

or

$$q_r = h_r A \; (T_o - T_i) \tag{43}$$

and

$$h_r = \sigma \epsilon \; T_o^3$$

For radiation to space, T_i will always equal 0 in the equation for q_r.

Energy Storage and Heat Balance

The net heat flow into region 0 must be stored in that region causing the temperature of node 0 to rise

$$q_{net} \cdot \Delta\Theta = Q_{stored} = V_d \, \rho \, C_p \, (t_o' - t_o) \qquad (44)$$

where the superscript is used to designate the temperature at the end of a time interval $\Delta\Theta$. Other quantities are identified in the List of Symbols. A heat balance on the cross hatched zone shown in Figure 4 gives

$$\Sigma q_{io} = \frac{C_o \, (t_o' - t_o)}{\Delta\Theta} \qquad (45)$$

where C_o is the heat capacity defined, for a unit depth, by the following equation;

$$C_o = V_o \rho \, C_p \qquad (46)$$

There will be a region for which the heat capacity will be assigned to each particular node 0. The area of the region is designed as V_o. The method for computing the area of the region associated with nodes in rectangular, square and annularly segmented subdivisions, are summarized in Table II. The equations for a triangular network are given in Reference (1). A rule to remember in solving for V_o is that the area V_o is bounded by the perpendicular bisectors of the flow paths to node 0. Once the area associated with a particular node has been determined, the heat capacity is computed from Equation 46.

By expanding Equation (45) we obtain

$$K_{1,0}(t_i - t_o) + K_{2,0}(t_2 - t_o) + \text{------} K_{n,o}(t_n - t_o) \qquad (47)$$

$$= \frac{C_o}{\Delta\Theta} (t_o' - t_o)$$
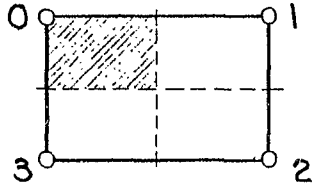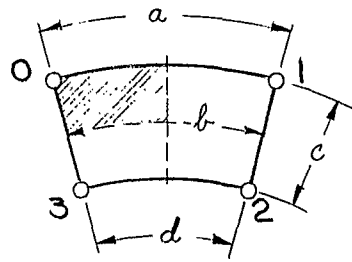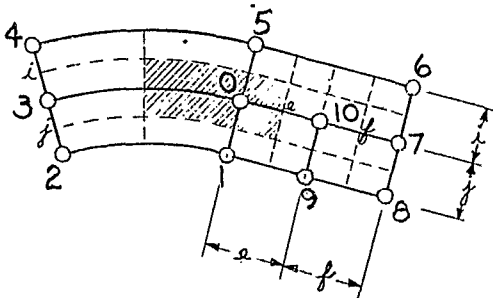
Equation (47) is strictly true for convection and conduction only; therefore, to include radiation all temperatures will be expressed on the absolute scale. Thus,

$$T_o' = \frac{K_{1,0} \, \Delta\Theta}{C_o} + \frac{K_{2,0} \Delta\Theta}{C_o} + \text{----} \frac{K_{n,o} \, \Delta\Theta}{C_o} + (1 - \frac{\Sigma \, K_o \, \Delta\Theta}{C_o}) \qquad (48)$$

TABLE II
Calculation of Heat Capacity Area Vo

Note: The area Vo is bounded by the perpendicular bisectors of the flow paths from the node under consideration.

NETWORK	Vo
SQUARE OR RETANGLE 	$$V_O = \frac{a}{2} \times \frac{b}{2} = \frac{ab}{4}$$ (1/4 of the square or rectangle is assigned to the node)
ANNULAR SEGMENT 	$$V_O = \left(\frac{\frac{a}{2} + \frac{b}{2}}{2}\right)\frac{c}{2} = \left(\frac{a+b}{8}\right)c$$ NOTE: $b = \frac{a+d}{2}$
COMBINED ELEMENTS 	$$V_O = \frac{a+3b}{16}i + \frac{c+3b}{16}j +$$ $$\frac{e\,i}{4} + \frac{e\,f}{4}$$

where K represents coefficients for all temperature dependent heat flows. As was previously pointed out, the procedure for calculating K will depend on the mode of heat flow along the particular path in question.

Criterion for Stability

Equation (48) is the basic working equation for the solution of transient heat flow problems by the numerical technique considered herein. By applying this equation to every node in a system, it is possible to compute the temperature history of the system given the initial conditions and given the boundary conditions as a function of time.

The characteristic of this and similar "explicit" techniques which is most often pointed out by those critical of the method is the relatively short time intervals $\Delta\theta$ which must be used to insure stability of the calculations. By close examination of Equation (48) it can be seen that for

$$(1 - \frac{\Sigma\, K_o\, \Delta\theta}{C_o}) < 0 \tag{49}$$

$T_o{}'$ depends on T_o in a negative sense. This is impossible physically; therefore, the generally accepted criterion to insure stability is

$$\frac{\Sigma\, K_o\, \Delta\theta}{C_o} \leqq 1 \tag{49a}$$

This criterion must be satisfied for every node in the system. In formulating a computer program based on the above equations, arrangements should be made for the machine to make a check of stability at each node and for all time intervals. This is desirable since the values for K at each node will be varying with time in the general case.

Division of a System into Nodes

The term, system, as used in this report, is any region of space and its contents which is regarded as isolated for purposes of observation and analysis. The object is to determine the temperature distribution in a physical system in which heat transfer is taking place by calculating temperatures at certain pre-selected, discrete points or nodes. The first step in the numerical analysis is

to divide the system by a network of reference points or nodes with points at which temperatures are desired included among the reference points. Obviously, additional nodes may also be necessary particularly in areas where gradients are changing sharply. As was described above, the division of time and the system subdivision are related through the stability criterion.

The method described in this report can be applied to any geometric arrangement of nodes as long as the region assigned to each node and the heat flow paths to and from each node can be adequately described. Procedures for handling systems subdivided by a network of squares, rectangles, annular segments or any combination of these are given above; triangular networks are described in Reference (1).

A system should be divided as simply as possible using as few nodes as possible since each additional point adds to the time and labor of computation. The computer input information is usually simplified if a given subdivision can be repeated throughout a large portion of the system since the geometric characteristics need only be entered once in the input to the computer.

It was mentioned previously that the usual rules for addition of conductances can be applied to different materials in series. Although this is true, a much simpler method can be utilized if a judicious selection of the node positions is made. Consider two materials, 1 and 2, each having different material properties and having a common interface. Also, consider the heat flow from a node in material 1 at a distance y_1 from the interface, to a node in material 2 located at a position y_2 from the interface. Using the subscripts 1 and 2 to designate materials and (f) to represent the conductance across the interface, the heat flow by conduction from node 1 to node 2 is

$$q = \frac{A}{\frac{y_1}{k_1} + \frac{1}{f} + \frac{y_2}{k_2}} (T_2 - T_1) \tag{50}$$

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and the conductance K_k is

$$K_{1,2} = \frac{A}{\frac{y_1}{k_1} + \frac{1}{f} 2 \frac{y_2}{k_2}} \tag{51}$$

This value must be calculated and entered as input to the computer.

The simplier method is to position the nodes at the interface with node 1 considered as being in material 1 and node 2 in material 2. Heat transfer between the nodes is assumed to take place only by convection across the interface (this is reasonable since the interface can be assumed to absorb no heat). In this arrangement the simplier and straight forward methods of calculating conductances can be applied.

SUMMARY OF EQUATIONS AND RELATED INFORMATION

The following equations, which enter into the combined heat-transfer and aerodynamic heating program previously described, have been programmed for the IBM 7074 computer at ABL.

<u>Aerodynamic Heat Input</u>   Reference (2)

1. Boundary Layer Temperature

   (a) Laminar Flow

   $$T_{bl} = T_\infty \ (1 + 0.17 \ M_a^2)$$

   (b) Turbulent Flow

   $$T_{bl} = T_\infty \ (1 + 0.18 \ M_a^2)$$

2. Film Coefficient

   (a) Laminar Flow

   $$h = 0.332 \ \frac{k^*}{x} \ (R_e^*)^{\frac{1}{2}}(P_r^*)^{1/3}$$

   (b) Turbulent Flow

   $$h = 0.0296 \ \frac{k^*}{x} \ (R_e^*)^{0.8} \ (P_r^*)^{1/3}$$

Note: The character*, appearing as a superscript over a given property indicates that that particular property is to be evaluated at the reference temperature.

3. Reference Temperature

   (a) General

   $$T^* = T_\infty \ (0.42 + 0.032 \ M_a^2) + 0.58 \ T_w$$

   (b) Approximate - Laminar Flow

   $$T^* = T_\infty \ (1 + 0.131 \ M_a^2)$$

   (c) Approximate - Turbulent Flow

   $$T^* = T_\infty \ (1 + 0.137 \ M_a^2)$$

4. Atmospheric Properties

   (a) Thermal Conductivity

$$k^* = 0.753 \times 10^{-6} + 6.319 \times 10^{-9} \, T^*$$

Reference (10)

   (b) Viscosity

$$\mu^* = 0.231 \times 10^{-7} \left[\frac{(T^*)^{3/2}}{T^* + 216} \right]$$

Reference (11)

   (c) Specific Heat at Constant Pressure

$$C_p^* = 32.17 \left[0.367 - \frac{500}{T^* + 3500} \right]$$

Reference (12)

   (d) Velocity of Sound

$$a = 49.02 \, (T_\infty)^{\frac{1}{2}}$$

   (e) Density

$$\rho^* = \rho_\infty \left(\frac{T_\infty}{T^*} \right)$$

Reference (13)

where

$$\rho_\infty = \rho_o \, (1 - 6.89 \times 10^{-6} H)^{4.256}$$

Reference (9)

for

H $\leq$ 35,352 ft., and

$$\rho_\infty = \frac{1.32 \, \rho_o}{\exp \, 1.452 + \dfrac{H - 35,352}{20,950}}$$

Reference (9)

for   H > 35,352 ft.

5. Dimensionless Numbers

   (a) Reynolds Number

$$R_e^* = \frac{\rho^* \, V_s x}{\mu^*}$$

   (b) Mach Number

$$M_a = \frac{V_s}{a_s}$$

   (c) Prandtl Number

$$P_r^* = \frac{C_p^* \, \mu^*}{k^*}$$

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6. Atmospheric Temperature

Several equations were used to represent the temperature as a function of time for both the Standard and Standard Hot atmospheres. These are given in the Abstract of the Computer program.[14]

Heat Transfer   Reference (1)

1. Node, Temperature

$$T_o{}' = F_{1,o}T_1 + F_{2,o}T_2 + \cdots\cdots F_{n,o}T_n + F_{o,o}T_o$$

where

$$F_{i,o} = K_{io}\,\frac{\Delta\theta}{C_o}$$

and

$$F_{o,o} = 1 - \frac{\Sigma\,K_{io}\,\Delta\theta}{C_o}$$

2. Conductance Factors

(a) Conduction

$$K = kS$$

Note: The value S is a geometric factor depending upon the geometric division into nodes and must be entered as input to the computer. The area A, normal to the heat flow path, is bounded by perpendicular bisectors of adjacent paths. The factor S is found by dividing A by the length of the corresponding path.

(b) Convection or Radiation

$$K = hA$$

3. Heat Capacity

$$C_o = V_o\,\rho\,C_p$$

Note: The area V_o is bounded by the perpendicular bisectors of the flow paths leading from the node under consideration. This term must be submitted as input to the machine.

4. Net Heat Transfer to the Structure

    (a) Rate of Transfer (Aerodynamic Heating)

$$q_{(\theta)} = h(T_{b1} - T_w) - h_r(T_w - T_i)$$

    (b) Total

$$Q = \Sigma \, {}^{\theta}_{o} \, q_{(\theta)} \, \Delta\theta$$

5. Stability Check

    Computer automatically adjusts $\Delta\theta$ until

$$\frac{\Sigma \, K_{1o}\Delta\theta}{C_o} \lessgtr 1 \qquad \text{for all nodes and time increments.}$$

Thermal Properties

Materials thermal properties necessary for computing transient heat flow and structural response are input terms and can be entered into the program as a function of temperature. Some typical thermal property values for some commonly used insulation materials are given in Table III. These are considered as suitable for use in preliminary design and study work; however, for project design work and more detailed analyses involving extended temperature ranges, these values are not recommended.

Table IV is a listing of the latest thermal property data for Spiralloy which are known at the present time. Since work is being continued on the evaluation of this material, and since the properties obviously must depend on changing fabrication techniques and component materials, they are likely to change quite rapidly. In the past the Polaris Division has been the leader in obtaining Spiralloy thermal property data; it is suggested that they be contacted when a detailed thermal analysis of a Spiralloy structure is required.

Units

Many of the working equations involve constants which were determined for a given set of units. For example, to account for radiation all temperatures were expressed in degrees Rankine. Most of the terms entering heat transfer calculations have derived units; e.g., units for thermal conductivity could be

$$\frac{\text{Btu}}{(\text{sec.})(\text{sq.ft.})\,°F/\text{ft.}}$$, and can be accurately expressed using several combinations of the fundamental units. Therefore, to avoid confusion and to avoid the programming of numerous conversion factors, the equations and program in general have been set up for the so-called English system of units using Btu's, feet, and seconds as the units for heat, length and time, respectively. All input should be reduced to this one consistent system of units for calculation. The basic units, along with the derived units, are identifiable in the List of Symbols.

When transient heat transfer calculations are carried out independent of the aerodynamic heating equations and radiation effects, the above procedure concerning units need not necessarily be followed. In this case, all that is required is that a consistent set of units be used.

Table III

Room Temperature Thermal Properties of Some Commonly Used Insulation Materials

Material	Thermal Conductivity $\frac{Btu}{(sec.)(sq.ft.)(°F/ft.)}$	Density $\frac{lb.}{cu.ft.}$	Specific Heat $\frac{Btu}{(lb.)(°F)}$	Reference
Buna-S Rubber	7×10^{-5}	85.5	0.44	19
Thermo-lag	2.22×10^{-5}	90.0	0.298	20
Cellulose Acetate	3.8×10^{-5}	81.0	0.32	21
Cork (2755*)	1.46×10^{-5} **	35.6	0.44	18
Boric-Acid Filled NBR Phenolic	4×10^{-5}	77.4	0.40	18
ATJ Graphite	1.458×10^{-2}	105.5	0.243	22 & 18
RPD - 150	7.15×10^{-5}	109.2	0.29	23
Pyrolytic Graphite	5.55×10^{-2} (A-Plane) 3.33×10^{-4} (B-Plane)	125.0	0.232	24

* Armstrong designation

** The thermal conductivity of this type of cork falls quite rapidly up to a temperature of approximately 180° at which $k = 1.076 \times 10^{-5}$.

Table IV

Thermal Properties of Spiralloy

Temperature °F	Thermal Conductivity[16] $\dfrac{Btu}{(sec.)(sq.ft.)(°F)}$	Density[17] $\dfrac{lbs.}{(cu.ft.)}$	Specific Heat[18] $\dfrac{Btu}{(lb.)(°F)}$
80	7.70	124	0.248
120	7.92	*	*
160	8.14		
200	8.37		
240	8.61		
280	8.88		
320	9.12		

* Data not available as a function of temperature.

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APPENDIX

ILLUSTRATIVE EXAMPLES

The following examples have been included to illustrate the method of solution of heat transfer problems by the procedures presented above and to point out the capabilities of the computer program. The actual preparation of data for machine input is explained in more detail in Reference 14. In presenting the results for each example, an attempt is made to point out particular program characteristics relating to that particular type of problem.

Example I is a typical cooling problem for which a "semi-analytical" solution is readily available. The results from the computer program are compared graphically with the solution taken from charts which correspond to the analytic solution and which are presented in the literature[15]. A favorable comparison is seen to result for this particular problem. Since the intent of this program was not to make a comprehensive study of the various aspects of heat transfer but to provide a simple tool for aerodynamic heating analysis, a more detailed study and further comparisons do not appear necessary at this time.

Example II is representative of several aerodynamic heating problems for which this program is ideally suited. In addition to pointing out the type of information available from the computer, this example also serves to illustrate the effects of changing cross-sections and the importance of a two-dimensional analysis in this type problem. Two approximate methods for determining film coefficient are also compared with the suggested method.

Example III was included to show the comparison of computed temperatures with those obtained from an actual flight test. Since most of the information pertaining to this flight is classified, only the temperature results are presented.

EXAMPLE I

Cooling of an Infinite Slab

A large slab of unalloyed titanium 4.25 inches thick is held at 1500°F before being hung vertically in 70° air to cool. A fan forces air over the slab at a velocity such that the convective film coefficient is maintained at a constant 62.7 Btu/(sq.ft.)(hr.)(°F). It is desirable to know the surface and midplane temperature as a function of time until the surface has dropped below 500°F.

(a) Consider average physical properties and no heat loss due to radiation.

(b) Compute new temperature-time relationships if radiation is considered. (Let $\epsilon = F_a \times F_e = 0.7$.)

(c) Determine temperature vs. time considering variation of thermal properties and accounting for radiation.

Solution

Since heat is being transferred at equal rates from both sides of the slab, only the temperatures on one side of the midplane are computed. The slab is divided into 9 elements as shown in the following sketch:
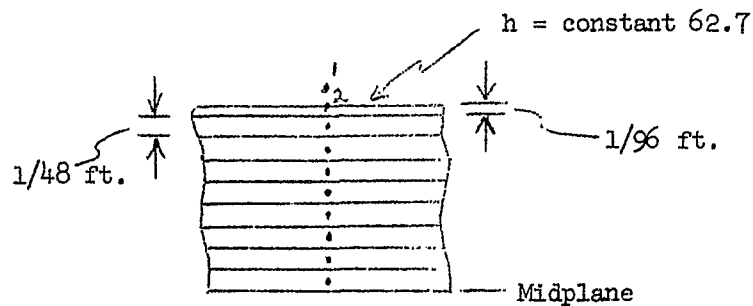
Each element is 1/48 ft. thick by 1 ft. square except for the element adjacent to the surface which is 1/96 ft. thick. This results in a total of 9 nodes in the metal plus 1 in the boundary layer. These are numbered consecutively from the outside inward starting at node 1 in the boundary layer. Node 10, although not exactly at the midplane, is assumed to represent midplane temperature.

A listing of the pertinent property values is given below:

Temperature	Thermal Conductivity	Specific Heat
80	10.1	0.125
200	9.8	0.130
400	9.72	0.136
600	9.82	0.143
800	10.0	0.151
1000	10.5	0.160
1200	11.1	0.171
1400	11.9	0.183
1510	12.4	0.19

The density ρ = 282 was assumed constant throughout the range of temperatures.

The thermal properties for solving parts (a) and (b) of this example were taken at an average temperature of approximately 1200°F.

The following data were compiled for computer input:

$$h = 62.7 \ \frac{Btu}{(hr.)(sq.ft.)(°F)}$$

$$k = 11.1 \ \frac{Btu}{(hr.)(sq.ft.)(°F/ft.)}$$

ρ = 282 lbs/cu.ft.

$$C = 0.158 \ \frac{Btu}{(lb.)(°F)}$$

$\Delta \theta$ = 0.002 hr.

T_{b1} = 70°F

T_i (initial temperature, all nodes) = 1500°F

$\epsilon = F_a \times F_e$ = 0.7 (for radiation part of the problem)

θ = 0.16 hr. (This value was taken from the analytical solution; however, the computer can be made to stop when a particular node has reached a certain temperature, if desired.)

Print out frequency - Arbitrarily selected as every tenth time increment.

V_0 = Nodal volume

V_2 = (1)(1)(1/96)

V_0 for nodes 3-10 is 0.02083 cu.ft.

S = Cross-sectional area of the path between nodes divided by the path length.

S_{12} - Mode of heat transfer is convection; cross-sectional area

(A_{12} = 1) required.

S_{23} and all others - Mode of heat transfer is conduction and S = 48 ft.

The heat transfer program was set up to conserve computer storage space by making possible the division of a system into five separate regions, each of which could be further divided by the nodal network into a maximum of 50 nodes. As a result, it is possible to enter data for an entire region without having to submit repetitious data for all the nodes in that region. In general, therefore, it is wise to lump all adjacent nodes having similar input information into a single region. Each region is ordinarily connected to adjacent regions by conduction between nodes at their common boundary.

Another secondary advantage arose out of the above method of programming. By not connecting the regions in a given run, it becomes possible to run five different problems at one time, submitting only the data for each problem (or region) which is different from the other regions. In the example now under consideration, parts (a), (b), and (c) were all run at one time and took approximately 1 minute including print-out.

The results for Example I are given in the figures which follow. Figure A-1 shows the surface temperatures as a function of time for the three different sets of conditions which were specified. Figure A-2 presents the same information but for the midplane temperature. For each of the two positions and for average material properties, temperatures were also computed from Shack Charts[15] which

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are based on analytical solution of the governing differential equations. The results are shown plotted on Figures A-1 and A-2, and are seen to be in quite close agreement with the numerical solutions.

In preparing input for this problem, a time increment of 0.002 hours, known to be unstable for the particular subdivision used, was selected. The computer automatically reduced the time increment by cutting it in half until a stable value ($\Delta \theta = 0.0008$ hours) was reached.

EXAMPLE II

Aerodynamic Heating of a Hypothetical Air-to-Air Missile

Figure A-3 shows the Mach number and altitude versus flight time for a hypothetical air-to-air missile. The trajectory shown is assumed to follow 15 minutes of flight at Mach equals 1.47 at 40,000 feet on a hot-atmosphere day. The following information is to be computed for the propulsion unit which is shown schematically in Figure A-4.

(1) h vs. θ at x = 5.5 using the approximate method (T_{bl} is assumed equal to T_w).

(2) h vs. θ at x = 3.5, 5.5 and 7.5 taking into account the computed wall temperatures.

(3) T_{bl} vs. θ

(4) Propellant-insulator interface temperature vs. time at x = 3.5, 5.5 and 7.5.

(5) q_{net} vs. θ at x = 3.5, 5.5 and 7.5.

(6) Surface temperature vs. length at θ = 30 sec. and 70 sec.

(7) Temperature distribution at x = 3.5 and 5.5 when the surface temperature at those points is maximum.

Solution

Figure A-5 illustrates how the rocket is divided into regions for analysis. Further division of the regions into nodes is shown in Figure A-6. In general, the entire length of the rocket is divided into three regions which are further divided into elements four inches long. Nodes are arranged to provide temperatures at a minimum of three depths in the chamber wall and two in the insulation. For simplification, only that portion of the rocket shown is considered in the analysis and all boundaries except those subjected to aerodynamic heating is assumed to be perfectly insulated. In cases where a more detailed analysis is required, the entire structure plus propellant should also be taken into account; however,

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it is not considered necessary in carrying out the intended purpose of this example.

The required thermal properties for Buna-S and Spiralloy rubber are taken from Table III and Table IV, respectively. The contact coefficient at the chamber-insulator interface is assumed to be $0.05 \frac{Btu}{(sq.ft.)(sec.)(°R)}$.

The results obtained from the computer are presented in the following figures. It is obvious that the calculated temperatures are much too high for proper functioning of the rocket and, in a real application, it would be necessary to add insulation to reduce the temperature of the critical components.

Figure A-7 illustrates the error involved in using the approximate methods of calculating heat transfer coefficients for aerodynamic heating. The range of Mach numbers and altitudes considered in this example are such that the differences are probably not critical; however, the margin of difference becomes larger as Mach number and altitude are increased. It should be noted, however, that the resulting error in computing temperatures from the approximate film coefficients will be smaller, percentage wise, than the error in the coefficients.

Figure A-8 demonstrates the variation in film coefficient with time and position. The coefficients are highest at positions which are the greatest distance from the leading edge. Another difference arises due to the variation in surface temperature along the length. The reference temperature at which the transport properties are evaluated for computing the film coefficient is a function of the surface temperature and the boundary layer temperature; therefore, the heat-transfer coefficient changes with changing surface temperature. The boundary layer temperature, shown in Figure A-9, is not a function of axial distance.

Figures A-10 through A-12 show temperatures at selected times and positions along the length and through the thickness of the rocket. Figure A-13 represents the heat rate at three positions along the length. These figures contain only a

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small portion of the data which were generated in solving the example.

Table V shows the output listing which is given on the first and last pages of each run. Table VI is an example of the data printed out at one of the thirty-six times which were called for in this example.

TABLE V

HERCULES POWDER COMPANY
ABL DATA PROCESSING DEPARTMENT   SCIENTIFIC GROUP — J. R. WISIALOWSKI

OUTPUT FOR PROG — 04-038 AERODYNAMIC HEATING AND/OR TWO-DIMENSIONAL HEAT TRAN.

CASE IDENTIFICATION — TRAJECTORY 1 TEST CASE — FOR ABSTRACT

TIME INCREMENT	INITIAL FREQUENCY	CONSTANT H
0.100000	20	-0.00000

AERODYNAMIC HEATING

STANDARD HOT ATMOSPHERE

ALTITUDE-MACH CURVE
VARIABLE

CALCULATED WALL TEMPERATURE

X-VALUES AT WHICH FILM COEFFICIENT IS CALCULATED

3.500
5.500
7.500

OUTPUT FOR PROG — 04-038 AERODYNAMIC HEATING AND/OR TWO-DIMENSIONAL HEAT TRAN.

CASE IDENTIFICATION — TRAJECTORY 1 TEST CASE — FOR ABSTRACT

TIME	70.10	ALT.-FT.	1300.0	MACH NO.	0.850

NODE			TEMPERATURES-DEG. F		
	REGION 1	REGION 2	REGION 3		REGION 4
TOT.(BTU/FT(2))	2.77E 02	1.69E 02	2.75E 02		

TIMES AND MAXIMUM TEMPERATURES FOR TH NODES

28.00	677.68	28.30	796.94	28.70	627.35

40

TABLE VI

TIME	30.00	ALT.-FT.	21000.0	MACH NO.	3.000	PAGE ( )

NODE					TEMPERATURES-DEG. F		
		REGION 1		REGION 2		REGION 3	
	H	2.3222E-02		2.0525E-02		2.0203E-02	
1	+1/3 FT.	791.8	SURFACE	813.8	−1/3 FT.	749.4	
2	TW NODE	672.1		808.7	TW NODE	626.4	
3	−1/3 FT.	817.7		803.2	+1/3 FT.	755.5	
4	.075 DEEP	790.7		798.6	.075 DEEP	747.1	
5	.075 DEEP	662.8	TW NODE	794.2	.075 DEEP	615.5	
6	.075 DEEP	817.7		789.7	.075 DEEP	753.7	
7	INTERFACE	789.0		785.5	INTERFACE	744.8	
8		653.5		781.5		605.1	
9		817.5		777.6		752.3	
10		644.3	.075 DEEP	813.4		595.2	
11		817.4		808.1		751.9	
12	INTERFACE	637.0		802.5	INTERFACE	587.5	
13	INTERFACE	767.7		797.7	INTERFACE	721.9	
14	INTERFACE	602.8		793.2	INTERFACE	553.9	
15	INNER SURF	764.3		788.6	INNER SURF	718.3	
16	INNER SURF	597.5		784.3	INNER SURF	548.8	
17	0.0 NODE	−459.7		780.2	0.0 NODE	−459.7	
18	TBL NODE	813.0		776.2	TBL NODE	813.0	
19	NOT USED	−459.7	INTERFACE	812.1	NOT USED	−459.7	
20	NOT USED	−459.7		806.8	NOT USED	−459.7	
21	NOT USED	−459.7		801.0	NOT USED	−459.7	
22	NOT USED	−459.7		796.2	NOT USED	−459.7	
23	NOT USED	−459.7		791.7	NOT USED	−459.7	
24	NOT USED	−459.7		786.9	NOT USED	−459.7	
25	NOT USED	−459.7		782.6	NOT USED	−459.7	
26	NOT USED	−459.7		778.5	NOT USED	−459.7	
27	NOT USED	−459.7		774.4	NOT USED	−459.7	
28	NOT USED	−459.7	INTERFACE	793.2	NOT USED	−459.7	
29	NOT USED	−459.7		787.5	NOT USED	−459.7	
30	NOT USED	−459.7		781.3	NOT USED	−459.7	
31	NOT USED	−459.7		776.2	NOT USED	−459.7	
32	NOT USED	−459.7		771.4	NOT USED	−459.7	
33	NOT USED	−459.7		766.3	NOT USED	−459.7	
34	NOT USED	−459.7		761.8	NOT USED	−459.7	
35	NOT USED	−459.7		757.4	NOT USED	−459.7	
36	NOT USED	−459.7		753.2	NOT USED	−459.7	
37	NOT USED	−459.7	INNER SURF	790.2	NOT USED	−459.7	
38	NOT USED	−459.7		784.4	NOT USED	−459.7	
39	NOT USED	−459.7		778.1	NOT USED	−459.7	
40	NOT USED	−459.7		773.0	NOT USED	−459.7	
41	NOT USED	−459.7		768.1	NOT USED	−459.7	
42	NOT USED	−459.7		763.0	NOT USED	−459.7	
43	NOT USED	−459.7		758.4	NOT USED	−459.7	
44	NOT USED	−459.7		754.0	NOT USED	−459.7	
45	NOT USED	−459.7		749.8	NOT USED	−459.7	
46	NOT USED	−459.7	0.0 NODE	−459.7	NOT USED	−459.7	
47	NOT USED	−459.7	TBL NODE	813.0	NOT USED	−459.7	

TBL,TW,Q RATE	18	2	3.27	47	5	0.38	18	2	3.77

EXAMPLE III

A3 POLARIS Second-Stage Motor Surface Temperature
Due to Aerodynamic Heating

The outside motor case temperature of the A3 POLARIS second-stage motor was calculated as a function of time at missile station 220 using A3X-2 flight trajectory data. These temperatures were compared with measured temperatures obtained at the same station during the A3X-2 flight. The measured outside motor case temperature data are compared with calculated values in Figure A-14. Due to the variation in reported values of thermal conductivity for Spiralloy, the calculations were carried out using two different values; results for both are presented for comparison in Figure A-14. It will be noted that reasonable agreement between the actual flight data and the calculated values was obtained.

The sudden temperature rise in the actual case temperature at a time of one second was caused by first-stage ignition and should not have had an effect on the peak temperature measured at 56 seconds.

Note: Points Correspond to "ANALYTICAL" solution taken from Schack Charts and using average properties.

"ANALYTICAL" Solution - Average Properties

Average Properties (Radiation Considered)

Average Properties (No Radiation)

Properties Varying with Temperature (Radiation Considered)

TEMPERATURE - DEGREES F.

TIME - HOURS
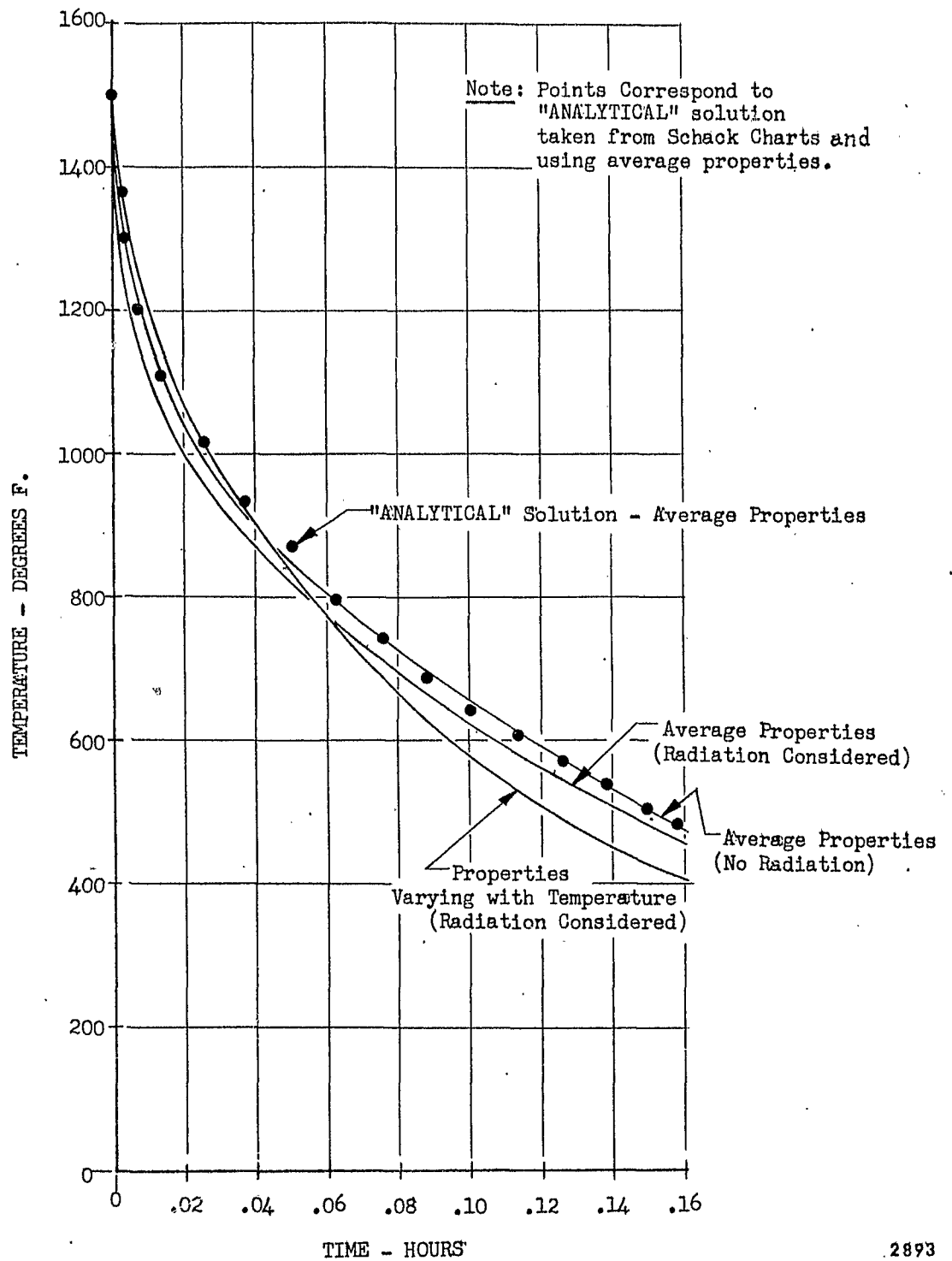
2893

FIGURE A1
Surface Temperature History

43
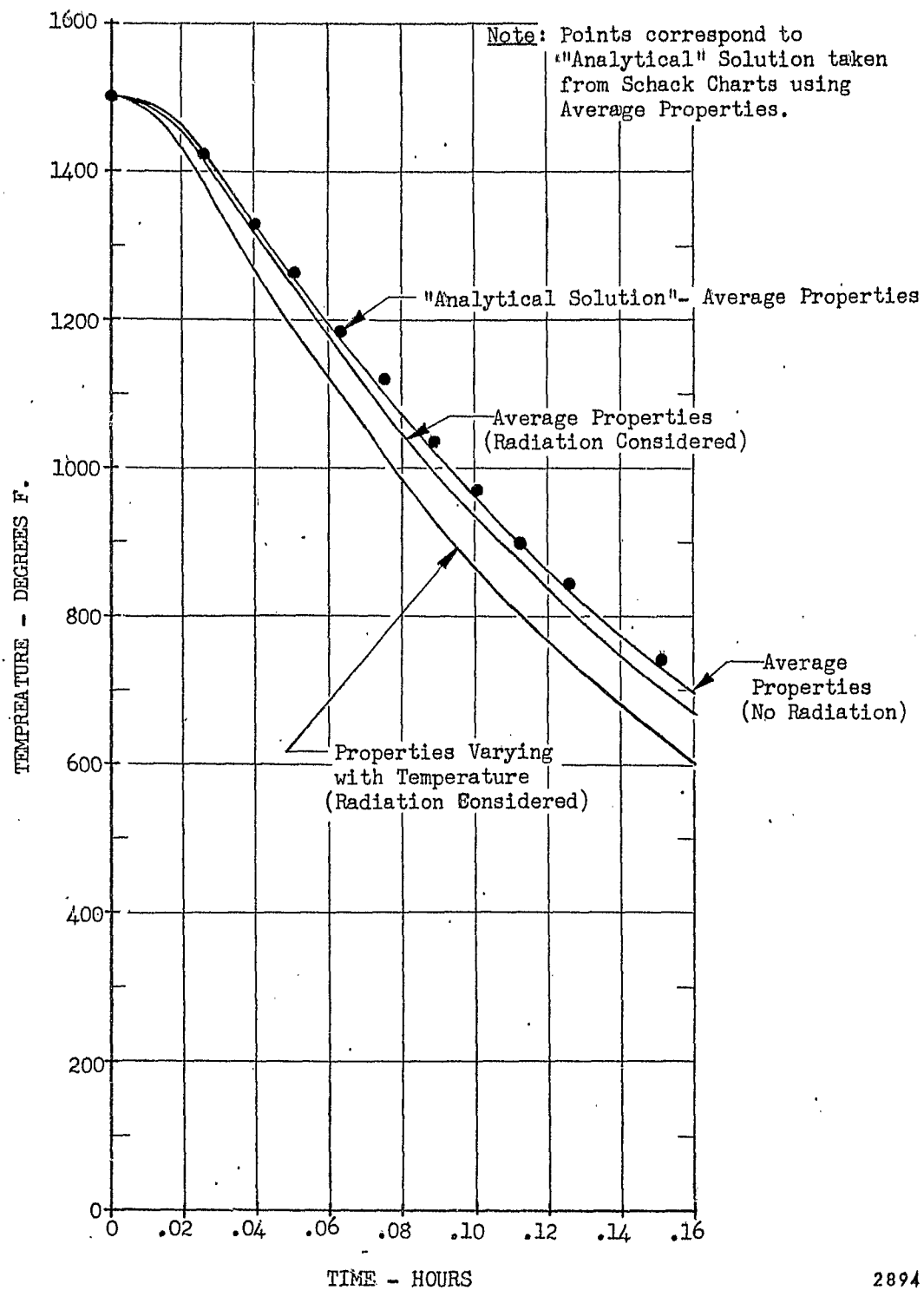
FIGURE A2
Mid-Plane Temperature

2894
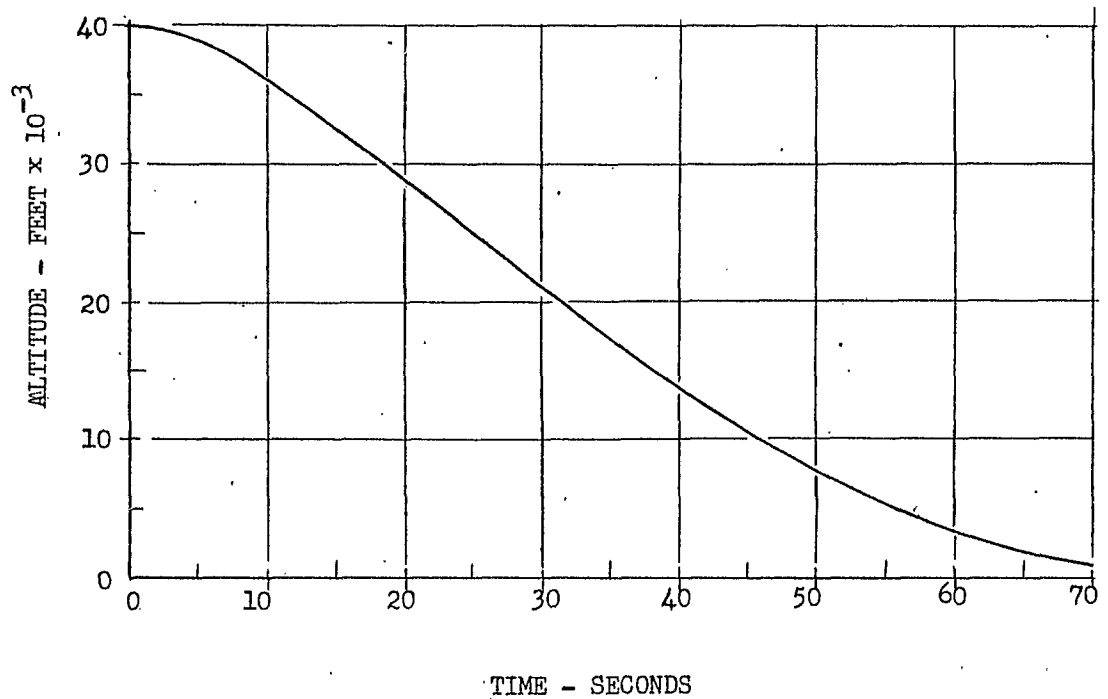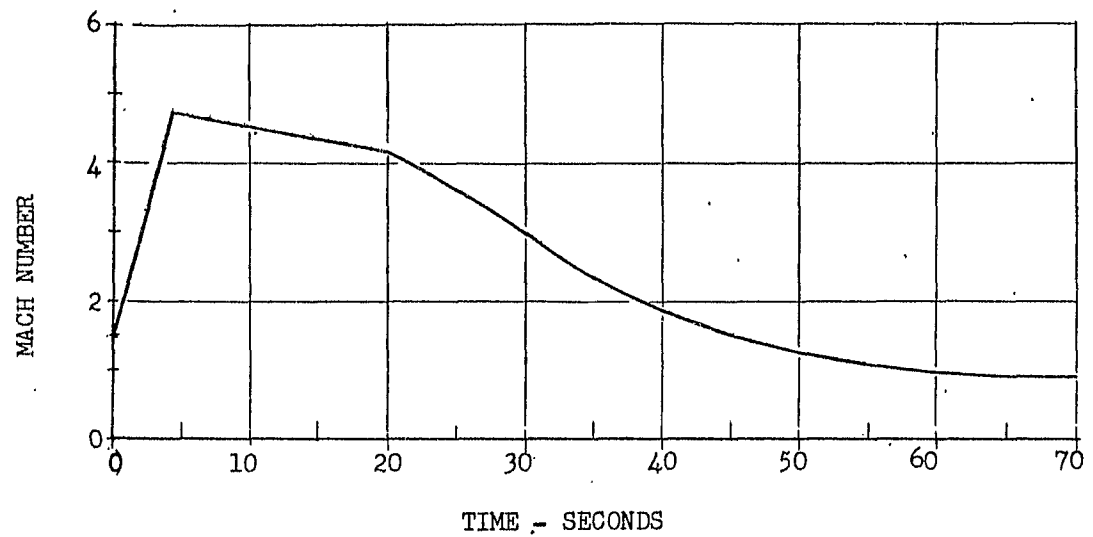
FIGURE A3
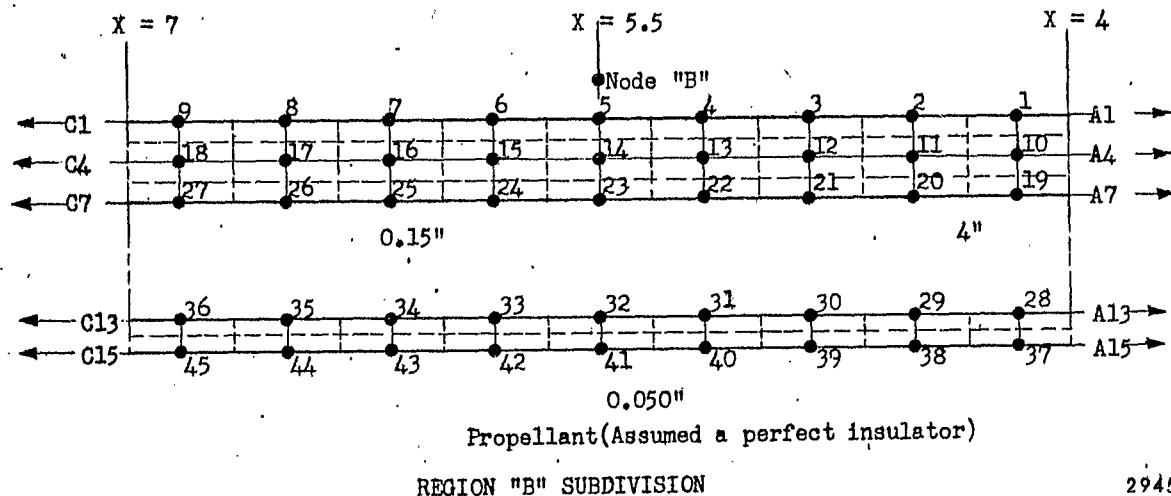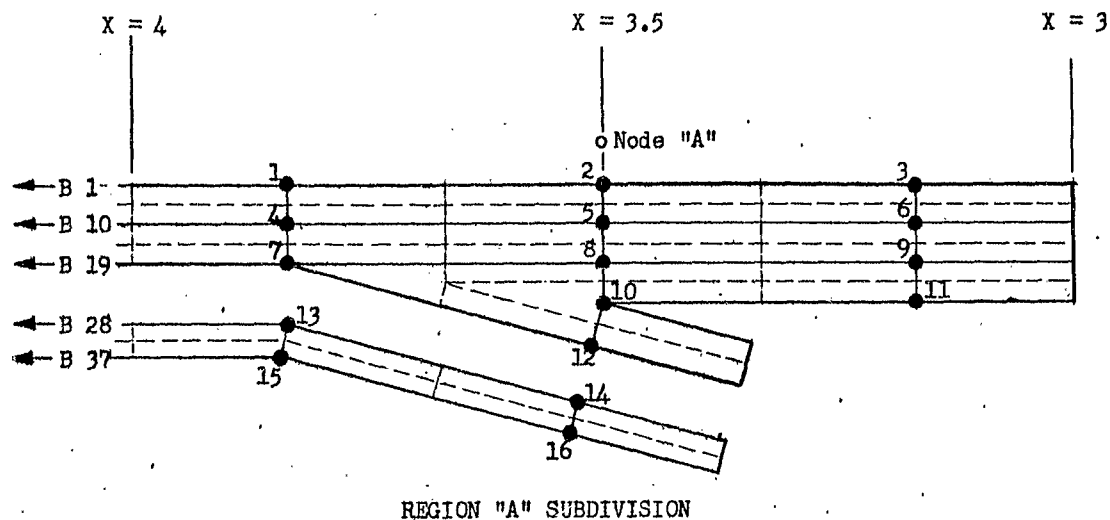Trajectory Information for a Hypothetical Air-to-Air Missile
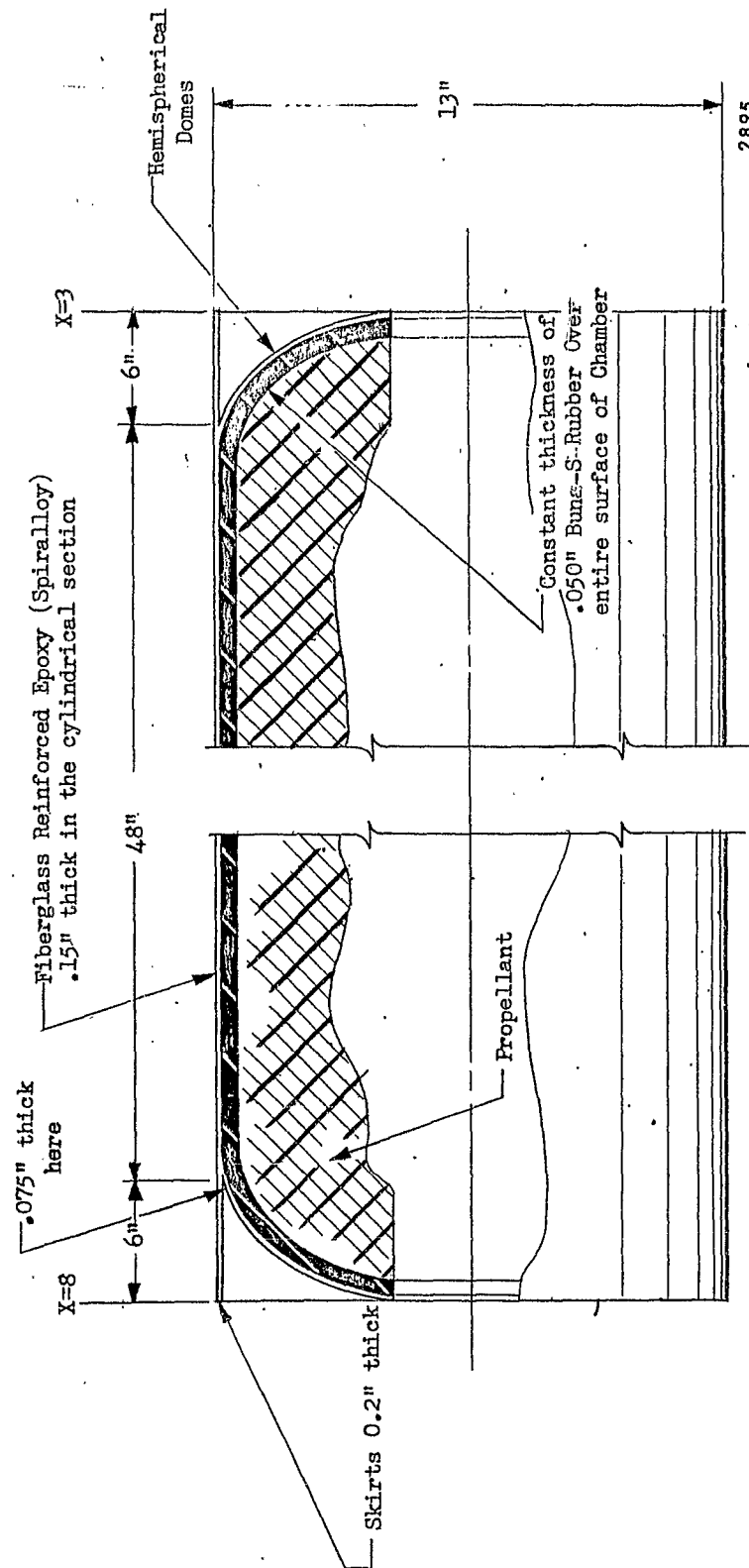
FIGURE A4

Schematic Representation of the Propulsion Unit for a Hypothetical Air-To-Air Missile
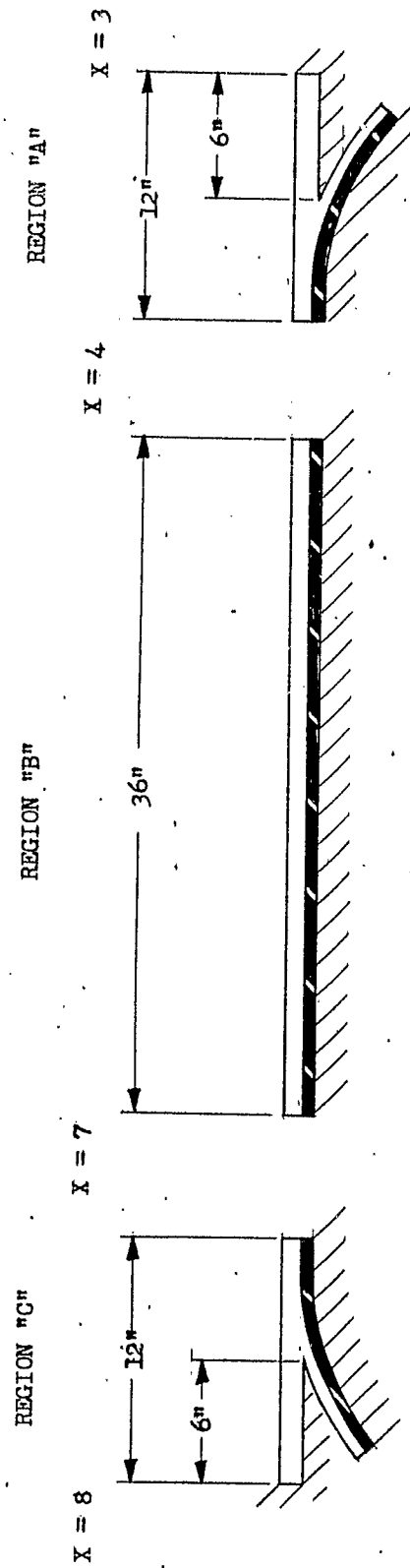
FIGURE A5

Division of Rocket Into Regions for Aerodynamic Heating Analysis

2904

47

REGION "A" SUBDIVISION

Propellant(Assumed a perfect insulator)

REGION "B" SUBDIVISION

2945

Region "C" Subdivision is a mirror image of Region "A".
Nodes 1,4,7,13 and 15 in Region "C" connect with nodes 9,18, 27,36, and 45, respectively, in Region "B".

FIGURE A6
Division of Regions into Nodes for Aerodynamic Heating Analysis
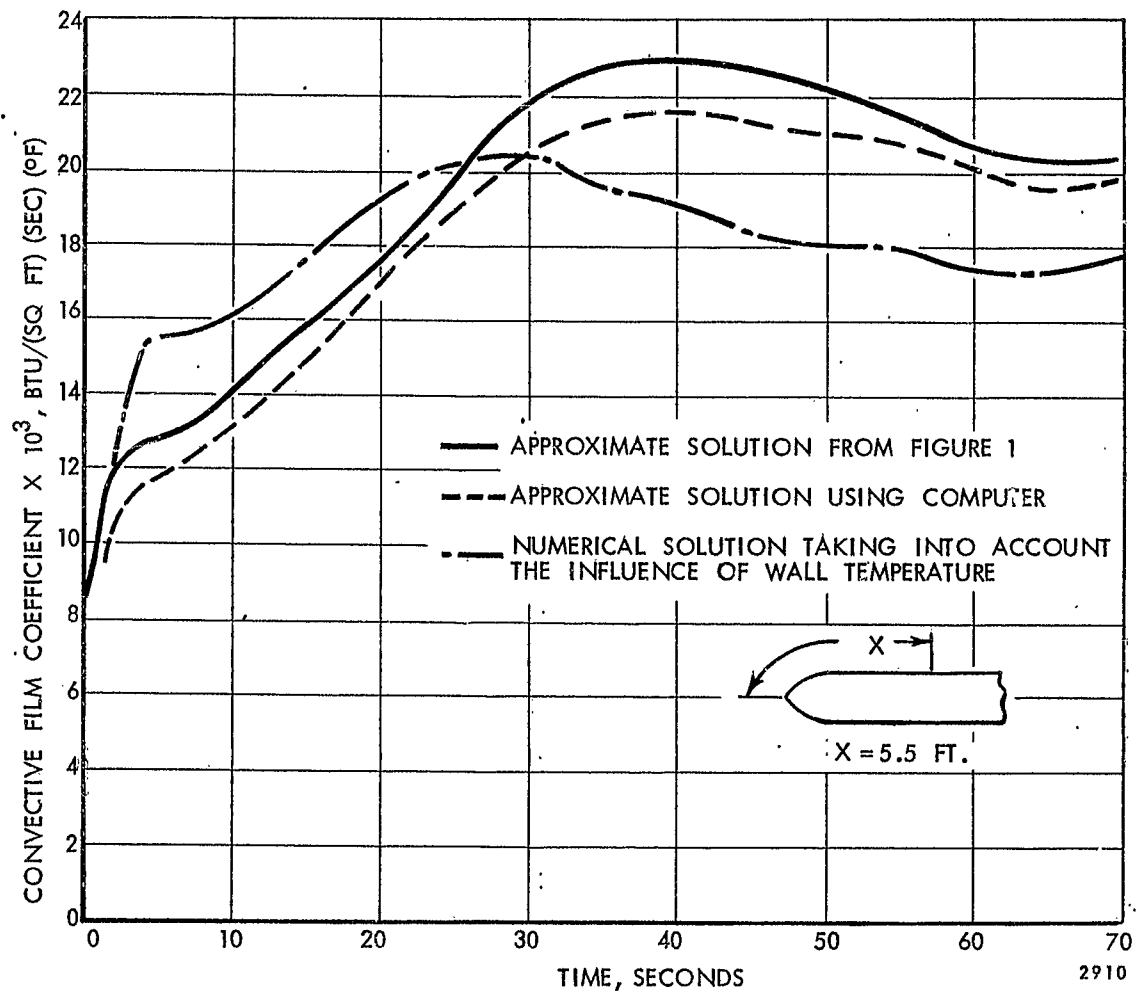
FIGURE A7
Comparison of Three Methods of Computing the Convective Film Coefficient for Aerodynamic Heating
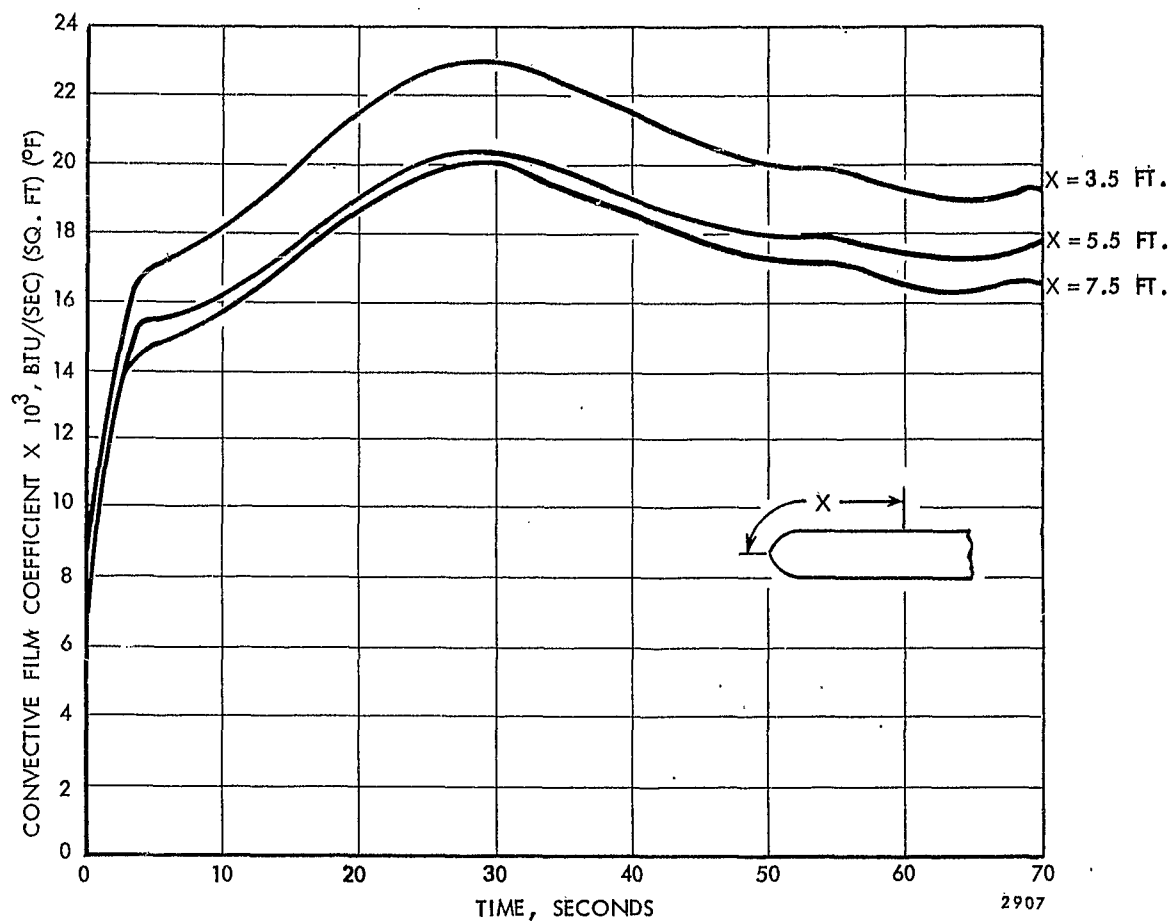
FIGURE A8
Film Coefficient at Three Positions Along the Axis of a Hypothetical Missile

50
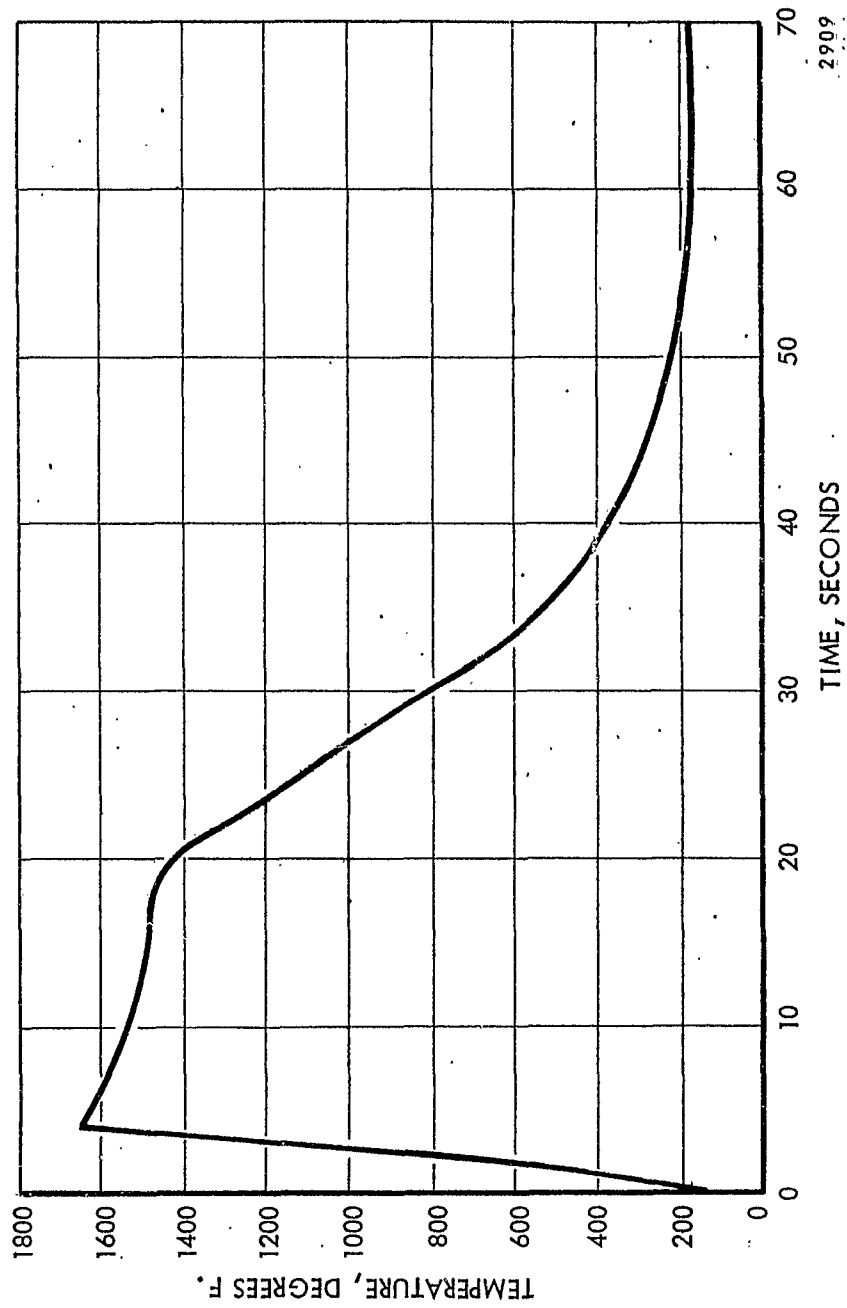
FIGURE A9
Boundary Layer Temperature for a Hypothetical Missile

2909

51
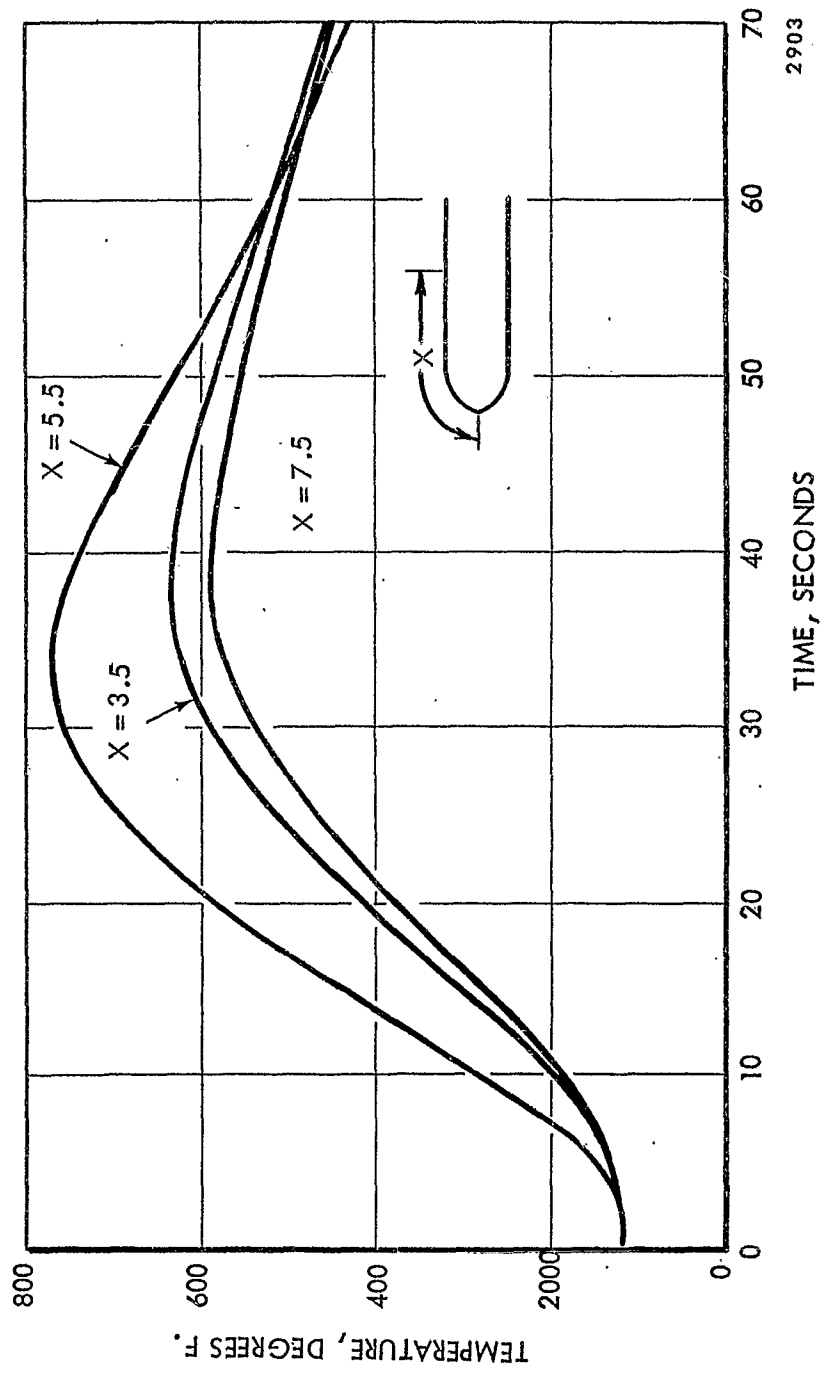
FIGURE A10
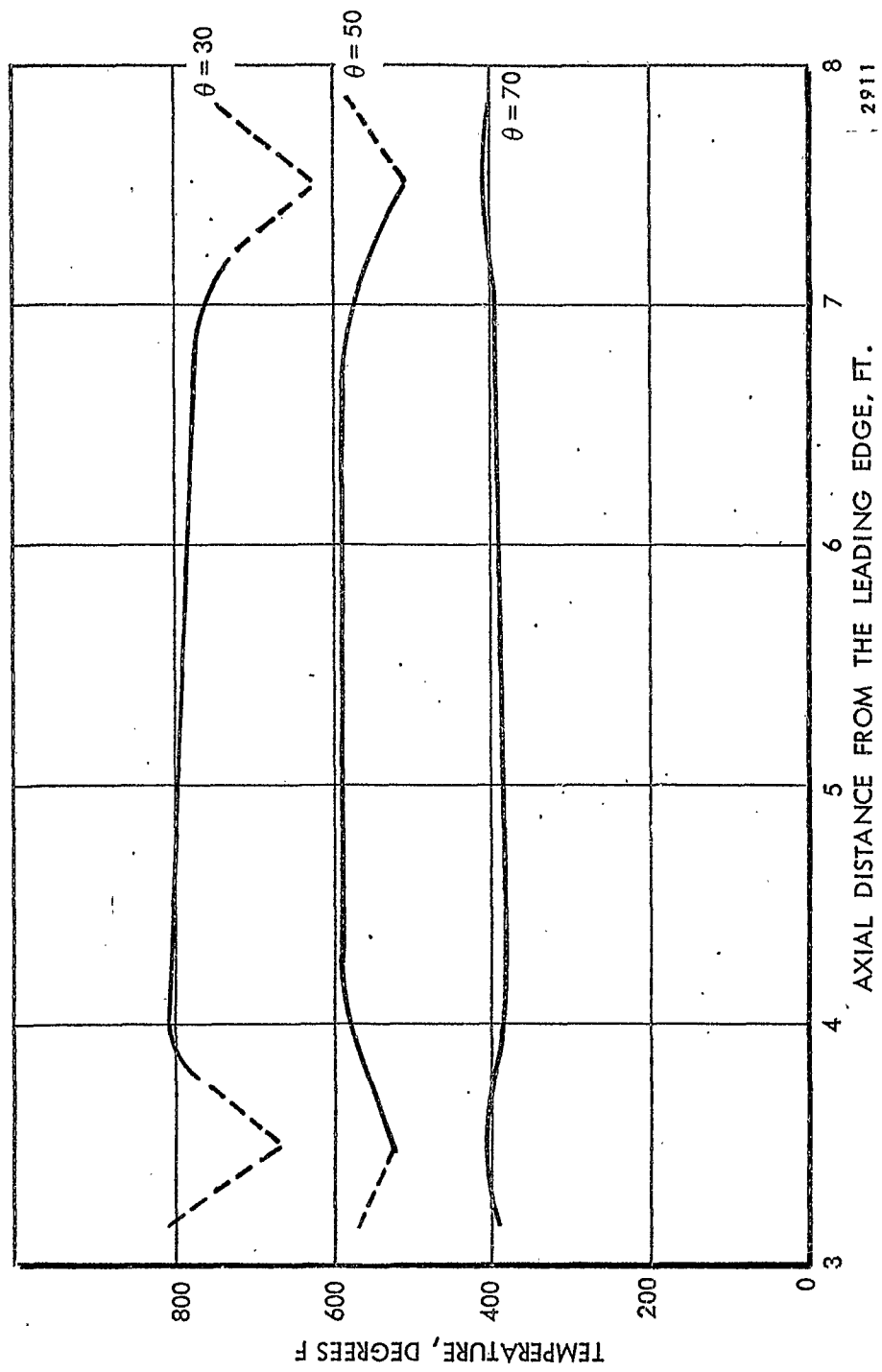—Propellant—Insulator Interface Temperatures

2903

52

FIGURE A11
Surface Temperatures

2911

FIGURE A12

Temperature Distribution Through the Thickness at the Time of Maximum Surface Temperature ($\theta \cong 28$ Sec.)
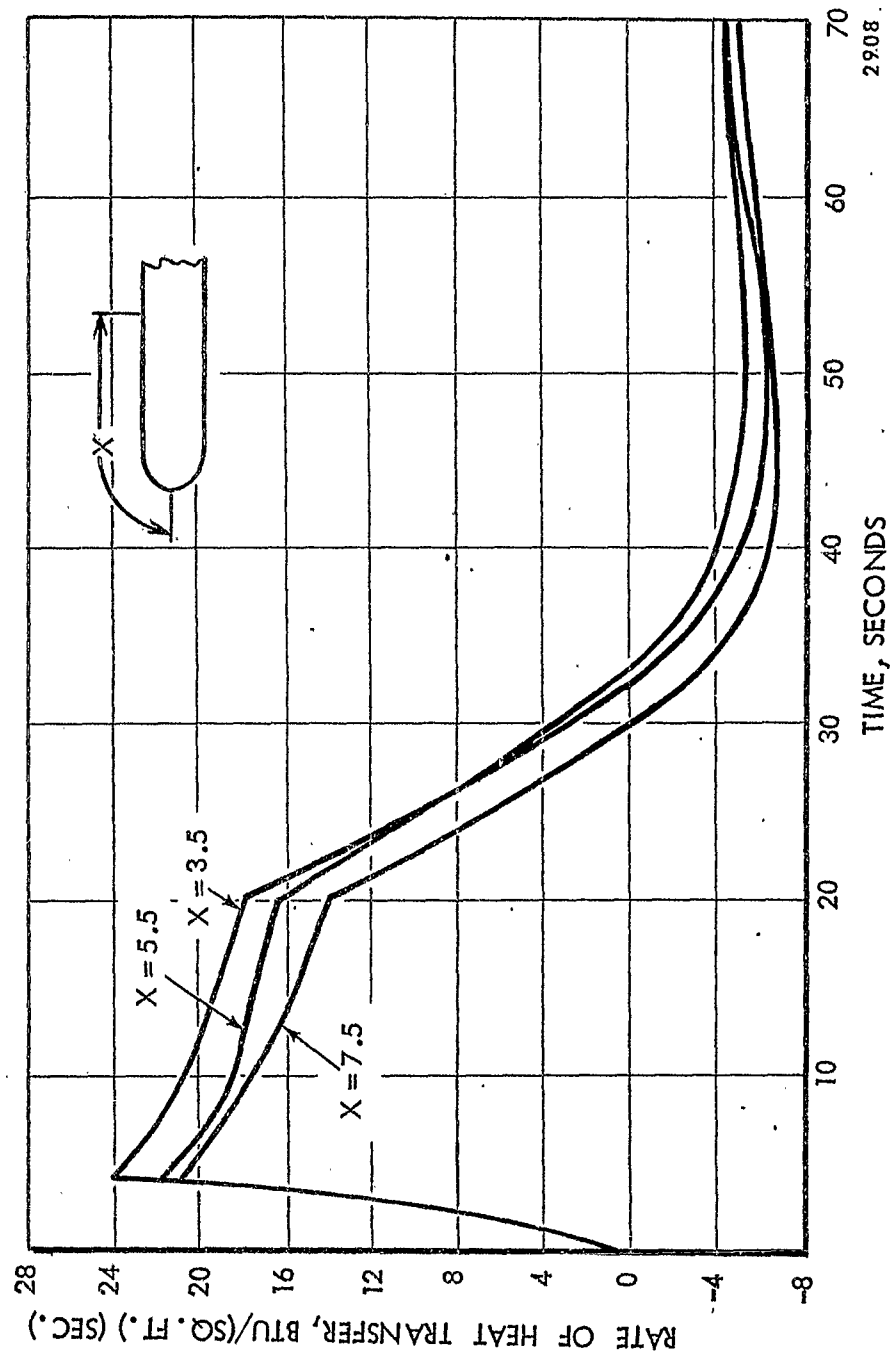
54

FIGURE A13

Rate of Heat Transfer at Three Positions Along the Axis of a Hypothetical Missile
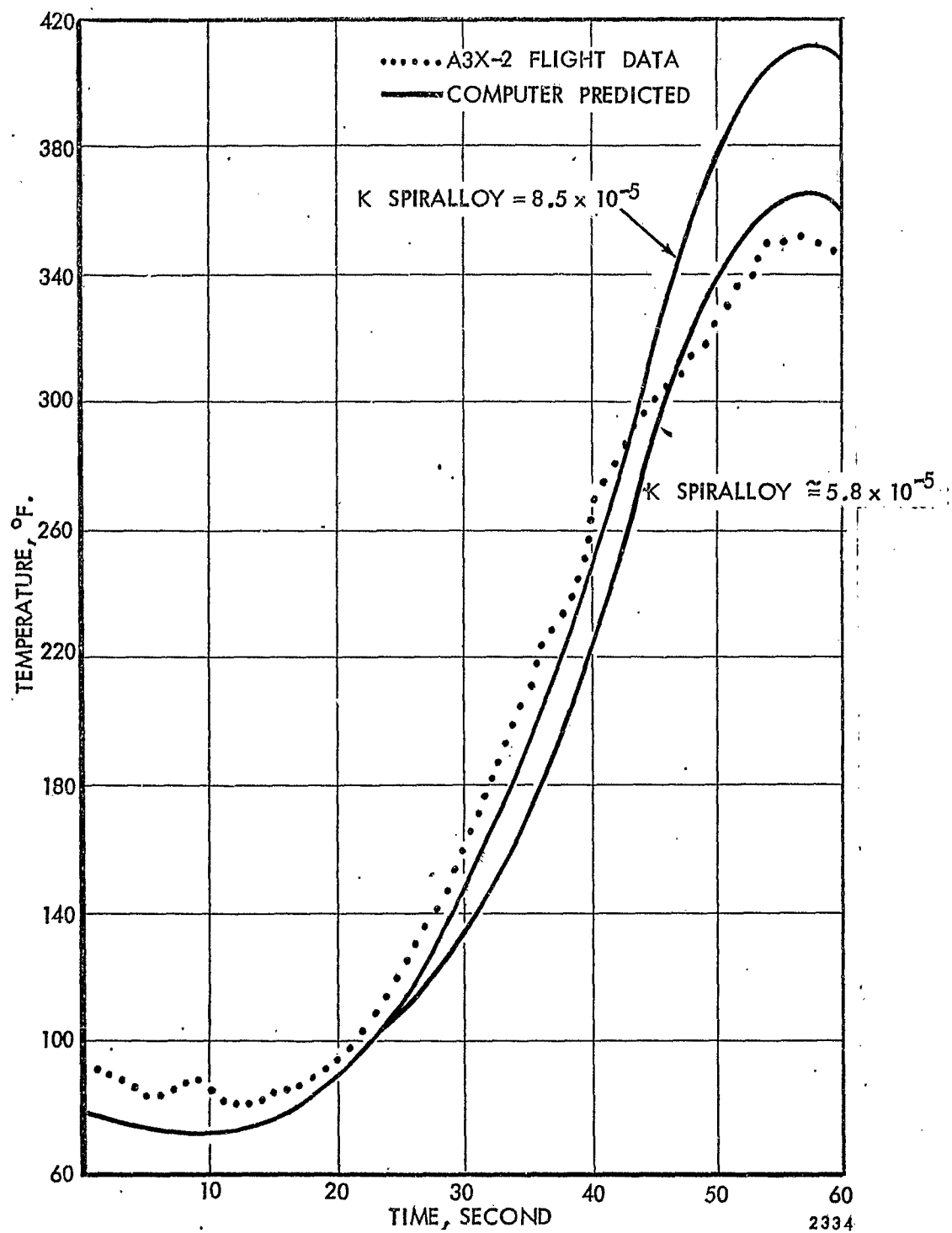
29.08.

FIGURE A14

Measured and Computed Motor Case Temperatures (MS 220) for Polaris Flight A3X-2